Solution of the Transport Equations for Two-Medium Slab Lattice with Anisotropic Scattering

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ABSTRACT

The transport equations for two-medium slab lattice are studied. The neutron density distributions and the disadvantage factor for thermal neutrons in water moderated, highly enriched uranium and uranium oxide fuel lattices were obtained with its external surface with reflection coefficient $R = 1$. Three coupled integral equations for the fluxes in the fuel and moderator are obtained. The Galerkin-method is used to solve the coupled integral equations for two-medium. The calculations are carried out for isotropic and anisotropic scattering in the moderator region and isotropic scattering in the fuel region. Numerical results are compared with the published calculations.

Key Words: The Disadvantage Factor / Two-Medium Slab Lattice / Galerkin Method / Reflection Coefficient.

INTRODUCTION

One of the fundamental problems in reactor physics is to determine the disadvantage factor. This factor is defined as the ratio of average neutron fluxes in the moderator to that in the fuel. In the research reactor, fuel is arranged in lumps of rods or plates separated by a material such as graphite, water or heavy water, in which neutrons are slowed. In many cases, the fuel elements surrounded by moderator (coolant) form a reactor lattice, which in the first step of reactor calculations is assumed infinite. In the plate unite cell concept its usually assumed the plate are infinite both y and z direction which reduces the problem of solution of the transport equation to a one dimensional one with constant flux boundary condition. With this assumption the plane unit cell does not need to be transformed. The transport and radiation problems in a two medium slab become very important and had been studied using different techniques (1-10). Attia (2) used transport equation in the moderator and fuel with assumption that the fuel is a uniform source of thermal neutrons.

In the present work, disadvantage factor was determined for water moderated, highly enriched uranium and uranium oxide fuel lattices from the thermal neutron density, obtained from solution of two neutron transport equations for inhomogeneous moderator region and homogeneous fuel region. A Galerkin method (7,8, 11) is used to solve coupled integral equations for two medium, and consequently the coefficients of the required functions for the two regions is obtained. Then calculations are carried out for isotropic and anisotropic scattering in the fuel and moderator respectively with different thickness and different reflecting coefficients.

BASIC EQUATIONS

Consider unite cell composed of a fuel rod surrounded by moderator. Neutrons, which escape from the fuel, will be slowed down in the moderator. The transport equation for an absorbing, emitting, with isotropic scattering is described for the fuel region as
The second transport equation with anisotropic scattering is described for the region where the neutron source arises from leakage from the moderator as

\[
(\mu \frac{\partial}{\partial z} + \Sigma_{t2})\phi_1(z,\mu) = \frac{\Sigma_{s1}}{2} \int_{-1}^{1} \phi_1(z,\mu') d\mu' = \frac{\Sigma_{s1}}{2} \phi_1(z),
\]

\[-1 \leq \mu \leq 1, \text{ and } -a \leq z \leq a
\]

The second transport equation with anisotropic scattering is described for the region where the neutron source arises from leakage from the moderator as

\[
(\mu \frac{\partial}{\partial z} + \Sigma_{t2})\phi_2(z,\mu) = \frac{\Sigma_{s2}}{2} \int_{-1}^{1} (1 + \alpha \mu \mu')\phi_2(z,\mu') d\mu' + Q(z)
\]

\[= G(z) + \frac{\Sigma_{s2}}{2} \phi_2^*(z), \quad a \leq z \leq b
\]

The boundary conditions are taken as

\[
\phi_1(-a,\mu) = \phi_1(-a,-\mu)
\]

\[
\phi_1(a,\mu) = \phi_2(a,\mu)
\]

\[
\phi_1(a,-\mu) = \phi_2(a,-\mu)
\]

\[
\phi_2(b,-\mu) = R\phi_2(b,\mu)
\]

With

\[
\phi_1(z) = \int_{-1}^{1} \phi_1(z,\mu) d\mu + \int_{0}^{1} \phi_1(z,-\mu) d\mu
\]

\[
G(z) = \frac{\Sigma_{s2}}{2} \phi_2(z) + Q(z),
\]

\[
\phi_2^*(z) = \int_{-1}^{1} \mu \phi_2^*(z,\mu) d\mu = \int_{0}^{1} \mu \phi_2(z,\mu) d\mu - \int_{0}^{1} \mu \phi_2(z,-\mu) d\mu
\]

Where \(\phi_1(z,\mu)\) is the angular flux at the position \(z\) and with the cosine of the direction of propagation, \(Q(z)\) is the source in the moderator, \(\sum_{s1}\) the scattering cross-section, \(\sum_{t2}\) the total cross-section, \(\phi_1(z)\) the total flux, \(\alpha\) the anisotropic scattering and \(R\) the reflection coefficient.

For the first medium, Eqs., is transformed by using the boundary conditions (5) and (6) to The integral form of eqs. (1) and (2) is obtained by integrating eq. (1) over \(z \in (-a,a)\) and eqs. (2) over \(z \in (a,b)\) as

\[
\phi_1(z,\mu) = \phi_1(-a,\mu) e^{-\Sigma_{t1}(a+z)/\mu} + \frac{\Sigma_{s1}}{2} \int_{-a}^{z} e^{-\Sigma_{s1}(z-z')/\mu} \phi_1(z') d\mu', \quad \mu \geq 0
\]
\[ \phi_1(z, -\mu) = \phi_1(a, -\mu) e^{\sum_{t_1(z-a)/\mu}} + \int_z^\mu \frac{e^{\sum_{t_1(z-z)}/\mu}}{\mu} \phi_1(z') \, dz', \quad \mu \geq 0 \]  

\[ \phi_2(z, \mu) = \phi_2(a, \mu) e^{\sum_{t_2(z-a)/\mu}} + \int_z^\mu \frac{e^{\sum_{t_2(z-z)}/\mu}}{\mu} \phi_2(z') \, dz', \quad \mu \geq 0 \]  

And

\[ \phi_2(z, -\mu) = \phi_2(b, -\mu) e^{\sum_{t_2(b-z)/\mu}} + \int_z^\mu \frac{e^{\sum_{t_2(z-z)}/\mu}}{\mu} \phi_2^*(z') \, dz', \quad \mu \geq 0 \]  

integrating eqs. (7-10) over \( \mu ? (0,1) \) and substituting in Eqs.(4-6) gives

\[ \phi_1(z) = \int_0^\mu [\phi_1(a, -\mu) e^{\sum_{t_1(z-a)/\mu}} + \phi_1(-a, \mu) e^{\sum_{t_1(z+a)/\mu}]} \, d\mu 
+ \frac{\Sigma_{t_2}}{2} \int_a^\mu E_1(\Sigma_{t_2}|z - z') \phi_1(z') \, dz' \]  

\[ G(z) = \frac{\Sigma_{t_2}}{2} \int_0^\mu [\phi_2(a, \mu) e^{\sum_{t_2(z-a)/\mu}} + \phi_2(b, -\mu) e^{\sum_{t_2(b-z)/\mu}]} \, d\mu 
+ \frac{\Sigma_{t_2}}{2} \int_a^\mu E_1(\Sigma_{t_2}|z - z') \phi_2^*(z') \, dz' + Q(z) \]  

\[ \phi_2^*(z) = \frac{\Sigma_{t_2}}{2} \int_0^\mu [\phi_2(a, \mu) e^{\sum_{t_2(z-a)/\mu}} + \phi_2(b, -\mu) e^{\sum_{t_2(b-z)/\mu}]} \, d\mu 
+ \frac{\Sigma_{t_2}}{2} \int_a^\mu E_1(\Sigma_{t_2}|z - z') \phi_2^*(z') \, dz' \]  

We rewrite Eqs. (6-10), at \( z = a, z = -a, z = b \) and \( z = a \) respectively, using the boundary conditions (3) and solving the system of the four resulting equations, we obtain

\[ \phi_1(-a, \mu) = \Delta^1 \left[ Y_4 e^{-\sum_{t_1(2a-a)/\mu}} + Y_2 + R e^{\sum_{t_2(2b-a)/\mu}} (Y_1 e^{-\sum_{t_2(2b-a)/\mu}} + Y_3 e^{-\sum_{t_2(b-a)/\mu}}) \right] \]  

\[ \phi_1(a, \mu) = \Delta^1 \left[ Y_4 + R e^{\sum_{t_2(2b-a)/\mu}} (Y_1 + Y_2 e^{-\sum_{t_2(2b-a)/\mu}} + Y_3 e^{-\sum_{t_2(b-a)/\mu}}) \right] \]
\[ \phi_2(b, \mu) = \Delta^1 R \left[ Y_1 e^{-\sum_{i=1}^2 (b-a)\mu} + Y_3 + e^{-\gamma\mu} (Y_4 e^{-\sum_{i=1}^2 2a\mu} + Y_2) \right] \] (16)

and

\[ \phi_2(a, \mu) = \Delta^1 \left[ Y_1 + e^{-\sum_{i=1}^2 a\mu} (R Y_3 e^{-\gamma\mu} + Y_4 e^{-\sum_{i=1}^2 2a\mu} + Y_2) \right] \] (17)

Where

\[ Y_1 = \frac{\Sigma_{Sl}}{2} \int_a^b \frac{e^{\Sigma_{i}(a-z')\mu}}{\mu} \phi_i(z') \, dz' \] (18)

\[ Y_2 = \frac{\Sigma_{Sl}}{2} \int_a^b \frac{e^{\Sigma_{l}(a-z')\mu}}{\mu} \phi_i(z') \, dz' \] (19)

\[ Y_3 = \int_a^b \frac{e^{\sum_{l} (b-z')\mu}}{\mu} G(z') \, dz' + a \frac{\Sigma_{Sl}}{2} \int_a^b e^{\sum_{l} (b-z')\mu} \phi_i(z') \, dz' \] (20)

and

\[ Y_4 = \int_a^b \frac{e^{\sum_{l} (z-a)\mu}}{\mu} G(z') \, dz' - a \frac{\Sigma_{Sl}}{2} \int_a^b e^{\sum_{l} (z-a)\mu} \phi_i(z') \, dz' \] (21)

with

\[ \Delta = (1 - \text{Re}^{-2y/\mu}), \quad \gamma = 2\Sigma_{i}\alpha + \Sigma_{l} (b - a) \]

Substituting the Eqs. (14-17) in Eqs. (11-13) gives

\[ \phi_1(z) = \frac{\Sigma_{Sl}}{4} \int_a^b \phi_1(z') K_i(z, z') \, dz' + \int_a^b G(z') K_i(z, z') \, dz' \]

\[ + \int_a^b \phi_2^*(z') K_i^2(z, z') \, dz' \] (22)

\[ G(z) = \frac{\Sigma_{Sl} \Sigma_{l2}}{4} \int_a^b \phi_1(z') K_2(z, z') \, dz' + \frac{\Sigma_{l2}}{2} \int_a^b G(z') K_2(z, z') \, dz' \]

\[ + \frac{\Sigma_{Sl}}{2} \int_a^b \phi_2^*(z') K_2^2(z, z') \, dz' + Q(z) \] (23)

\[ \phi_2^*(z) = \frac{\Sigma_{l2}}{2} \int_a^b \phi_1(z') K_3(z, z') \, dz' + \int_a^b G(z') K_3(z, z') \, dz' \]

\[ + \frac{\Sigma_{l2}}{2} \int_a^b \phi_2^*(z') K_3^2(z, z') \, dz' + Q(z) \] (24)

Where

\[ K_i(z, z') = RF_i \left[ Y - \Sigma_{i}(z + z') \right] + RF_i \left[ Y + \Sigma_{i}(z - z') \right] + E_i \left[ \Sigma_{i1}|z - z'| \right] \] (25)

\[ K_i^1(z, z') = RF_i \left[ \Sigma_{l2} (2b - a - z') + \Sigma_{i1} (a - z) \right] + RF_i \left[ \Sigma_{l2} (2b - a - z') + \Sigma_{i1} (a - z) \right] \]

\[ + (-1)^i \left[ F_i \left[ \Sigma_{l2} (z - a) + \Sigma_{i1} (a - z) \right] - F_i \left[ \Sigma_{l2} (z - a) + \Sigma_{i1} (a + z) \right] \right] \] (26)

\[ K_i^2(z, z') = RF_i \left[ \Sigma_{l2} (2b - a - z) + \Sigma_{i1} (a - z') \right] + F_i \left[ \Sigma_{l2} (z - a) + \Sigma_{i1} (a - z) \right] \] (27)
\[ K_2(z, z') = RF_1[\Sigma_{12}(z - z') + \gamma] + RF_1[\Sigma_{12}(2b - z' - z)] + E_1(\Sigma_{12}|z - z'|) + F_1[\Sigma_{12}(z' + z - 2a) + 2\Sigma_{11}(a)] + F_1[\Sigma_{12}(z' - z) + \gamma] \]  
\[ (28) \]

\[ K_2(z, z') = RF_2[\Sigma_{12}(z - z') + \gamma] + RF_2[\Sigma_{12}(2b - z' - z)] - F_2[\Sigma_{12}(z' - z) + \gamma] - F_2[\Sigma_{12}(z' + z - 2a) + 2\Sigma_{11}(a)] + \text{sign}(z - z')E_2(\Sigma_{12}|z - z'|) \]  
\[ (29) \]

\[ K_3(z, z') = F_2[\Sigma_{12}(z - a) + \Sigma_{11}(a - z')] - RF_2[\Sigma_{12}(2b - a - z) + \Sigma_{11}(a - z')] \]  
\[ (30) \]

\[ K_3(z, z') = RF_3[\Sigma_{12}(z - z') + \gamma] - RF_3[\Sigma_{12}(2b - z' - z)] - F_3[\Sigma_{12}(z' - z) + \gamma] + F_3[\Sigma_{12}(z' + z - 2a) + 2\Sigma_{11}(a)] + E_3(\Sigma_{12}|z - z'|) \]  
\[ (31) \]

\[ K_3(z, z') = RF_3[\Sigma_{12}(z - z') + \gamma] - RF_3[\Sigma_{12}(2b - z' - z)] + F_3[\Sigma_{12}(z' - z) + \gamma] - F_3[\Sigma_{12}(z' + z - 2a) + 2\Sigma_{11}(a)] + E_3(\Sigma_{12}|z - z'|) \]  
\[ (32) \]

\[ F_n(z) = \int_0^1 e^{-z^2/\mu} \mu^{n-2} (1 - \text{Re}^{2z^2/\mu}) \]  

and \( E_n(z) \) is the exponential integral function

\[ E_n(z) = \int_0^1 e^{-z^2/\mu} \mu^{n-2} d\mu \]

**SOLUTION METHOD**

According to the Galerkin method the trial functions approximating of the integral Eqs.(22-24) are expressed by the expansion forms

\[ \Phi_1(z) = \sum_{n=0}^{N} a_n z^n \]  
\[ (33) \]

\[ G(z) = \sum_{n=0}^{N} C_n z^n \]  
\[ (34) \]

\[ \Phi_2^*(z) = \sum_{n=0}^{N} D_n z^n \]  
\[ (35) \]

Where \( a_n, C_n \) and \( D_n \) are unknown coefficients.

Substituting these expansions into Eqs.(22-24), multiplying the resultant equations by \( z^m, m = 1, \ldots, N \) integrating Eq.(22) over \( z \in (-a, a) \) and Eqs.(23,24) over \( z \in (a,b) \) respectively, leads to

\[ \sum_{n=0}^{N} \{ a_n H_{11}^{11} - C_n H_{11}^{12} - D_n H_{11}^{13} \} = 0 \]  
\[ (36) \]

and

\[ \sum_{n=0}^{N} \{ a_n H_{21}^{21} + C_n H_{21}^{22} - D_n H_{21}^{23} \} = Q_m \]  
\[ (37) \]

\[ \sum_{n=0}^{N} \{ a_n H_{31}^{31} - C_n H_{31}^{32} + D_n H_{31}^{33} \} = 0 \]  
\[ (38) \]
with

\[ H_{nm}^{11} = \int_{a}^{b} z^{n+m} dz - \frac{\Sigma_{a}}{2} \int_{a}^{b} z^{m} \int_{a}^{b} z'^{n} K_{1}(z, z') dz dz' \]  

(39)

\[ H_{nm}^{12} = \int_{a}^{b} z^{m} \int_{a}^{b} z'^{n} K_{1}(z, z') dz dz' \]  

(40)

\[ H_{nm}^{13} = \frac{a \Sigma_{s}}{2} \int_{a}^{b} z^{m} \int_{a}^{b} z'^{n} K_{1}(z, z') dz dz' \]  

(41)

\[ H_{nm}^{21} = \sum \frac{\Sigma_{a}}{4} \int_{a}^{b} z^{m} \int_{a}^{b} z'^{n} K_{2}(z, z') dz dz' \]  

(42)

\[ H_{nm}^{22} = \int_{a}^{b} z^{n+m} dz - \frac{\Sigma_{s}}{2} \int_{a}^{b} x^{m} \int_{a}^{b} z'^{n} K_{2}(z, z') dz dz' \]  

(43)

\[ H_{nm}^{23} = \frac{a}{2} \left( \frac{\Sigma_{s}}{2} \right)^{2} \int_{a}^{b} z^{m} \int_{a}^{b} z'^{n} K_{2}^{2}(z, z') dz dz' \]  

(44)

\[ H_{nm}^{31} = \sum \frac{\Sigma_{a}}{2} \int_{a}^{b} z^{m} \int_{a}^{b} z'^{n} K_{3}(z, z') dz dz' \]  

(45)

\[ H_{nm}^{32} = \int_{a}^{b} z^{m} \int_{a}^{b} z'^{n} K_{3}(z, z') dz dz' \]  

(46)

\[ H_{nm}^{33} = \int_{a}^{b} z^{n+m} dz - a \frac{\Sigma_{s}}{2} \int_{a}^{b} z^{m} \int_{a}^{b} z'^{n} K_{3}^{2}(z, z') dz dz' \]  

(47)

and

\[ Q_{m} = \int_{a}^{b} z^{m} Q(z) dz \]  

(48)

The simultaneous solution of the set of Eqs.(36-38) will give the values of the coefficients \( a_{a}, C_{a}, \) and \( D_{a} \) and consequently, the required functions \( \Phi_{1}(z), \Phi_{2}(z) \) and \( G(z) \).

**RESULT AND DISCUSSION**

The above equations are used to calculate the average flux in the fission product \( \Phi_{1}(x) \) and in the moderator \( \Phi_{2}(z) \) and consequently, the disadvantage factor \( \zeta = (\Phi_{2}/\Phi_{1}) \) for two cases. The first we consider highly enriched uranium / water lattices, where \( \Sigma_{a} = 0.9209 \text{ cm}^{-1}, \Sigma_{s} = 0.7278 \text{ cm}^{-1} \) for highly enriched uranium and \( \Sigma_{a} = 0.00197 \text{ cm}^{-1}, \Sigma_{s} = 3.6713 \text{ cm}^{-1} \) in the moderator. Secondly, we consider uranium oxide / water lattices, where \( \Sigma_{a} = 0.169 \text{ cm}^{-1}, \Sigma_{s} = 0.372 \text{ cm}^{-1} \) for uranium oxide. In all cases the calculations are carried out for isotropic forward and backward linear anisotropic scattering by using five terms of expansion (36-38) with a unit source in the moderator.
Table (1): The average fluxes and disadvantage factor for isotropic, forward and backward scattering with $a = 0.1016$ cm and $b = 0.2997$ cm with different reflection coefficients in case highly enriched uranium / water lattices.

<table>
<thead>
<tr>
<th>R</th>
<th>Isotropic Scattering</th>
<th>Forward Scattering</th>
<th>Backward Scattering</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present</td>
<td>Ref.[10]</td>
<td>present</td>
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<tr>
<td>0.2</td>
<td>0.47744 0.47741</td>
<td>0.51252 0.51249</td>
<td>1.07347 1.07348</td>
</tr>
<tr>
<td>0.4</td>
<td>0.62670 0.62635</td>
<td>0.68014 0.68002</td>
<td>1.08527 1.08568</td>
</tr>
<tr>
<td>0.6</td>
<td>0.87691 0.87673</td>
<td>0.96098 0.96093</td>
<td>1.09587 1.09605</td>
</tr>
<tr>
<td>0.8</td>
<td>1.38662 1.38633</td>
<td>1.53161 1.53154</td>
<td>1.10456 1.10474</td>
</tr>
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</table>

The anisotropic scattering $\alpha$ may be given in terms of the Legendre polynomial coefficients $a_n^*$ in the form [12]

$$
\alpha = 2 \sum_{n=0}^{\infty} \frac{(-1)^n a_{2m+1}^* (2m)!}{[2^{2m+1} m!(m + 1)!]}
$$

And give linear anisotropic parameters $\alpha = 1.81517$ and - 0.58659, respectively. The calculations for the average fluxes and disadvantage factor in case of highly enriched uranium / water lattices are given in table (1) for $a = 0.1016$ cm$^{-1}$ and $b = 0.2997$ cm$^{-1}$ with different reflection coefficients. Table (2) and Figure 2 shows the result of the average fluxes $\phi_1(z)$, $\phi_2(z)$ and disadvantage factors for fixed moderator thickness of 1 m.f.p. and various fuel region thickness with different reflection coefficients. The same calculations are carried out for uranium oxide / water lattices, and are tabulated in tables (3-4). The effect of the reflection coefficient R is seen in all results which are compared with those of Ref.(2), which show good agreement with exact results. The Disadvantage factor and neutron flux for highly enriched uranium and uranium oxide shown in Figure 1,2 respectively. Table 2 and 4 shown in Figure 3 and 4.
Table (2): The average fluxes and disadvantage factor for anisotropic scattering using various fuel (highly enriched uranium) thickness and different values for $R$ with $b = 1$ m.f.p.

<table>
<thead>
<tr>
<th>A</th>
<th>$R = 0.5$</th>
<th>Forward scattering</th>
<th>Backward scattering</th>
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<tr>
<td></td>
<td>$\phi_1$</td>
<td>$\phi_2$</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>Present</td>
<td>Ref[10]</td>
<td>Present</td>
<td>Ref[10]</td>
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<tr>
<td>0.11</td>
<td>0.707185</td>
<td>0.760321</td>
<td>1.07514</td>
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<tr>
<td>0.15</td>
<td>0.685480</td>
<td>0.749192</td>
<td>1.09295</td>
</tr>
<tr>
<td>0.22</td>
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<td>0.734987</td>
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<tr>
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<td>0.725833</td>
<td>1.12515</td>
</tr>
<tr>
<td>0.30</td>
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<td>1.14084</td>
</tr>
<tr>
<td>0.35</td>
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</tr>
<tr>
<td>0.5</td>
<td>0.495547</td>
<td>0.587561</td>
<td>1.18568</td>
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Table (3): The average fluxes and disadvantage factor for anisotropic scattering using various fuel UO$_2$ (uranium Oxide) thickness and different values for $R$ with $b = 1$ m.f.p.

<table>
<thead>
<tr>
<th>R</th>
<th>Isotropic Scattering</th>
<th>Forward scattering</th>
<th>Backward scattering</th>
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<tr>
<td></td>
<td>$\phi_1$</td>
<td>$\phi_2$</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.53074</td>
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Table (4): The average fluxes and disadvantage factor for isotropic scattering using various fuel $UO_2$ (uranium Oxide) thickness and different values for $R$ with $b = 1$ m.f.p

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R=0.9

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R=1.0

Fig(1) Disadvantage factor in case highly enriched uranium and uranium Oxide for isotropic scattering with $a = 0.1016$ cm and $b = 0.2997$ cm for different reflection coefficients

Fig(2) The average fluxes for anisotropic scattering using various fuel thickness and different values for $R$ with $b = 1$ m.f.p
CONCLUSION

The average fluxes in the fuel and in the moderator and consequently, the disadvantage factor are calculated by solving the transport equation for two-medium slab lattice. The A Galerkin method is used to solve three coupled integral equations for two medium, and consequently the coefficients of the required functions for the two regions is obtained. The calculations for an isotropic, forward and backward linear anisotropic scattering scattering, inhomogeneous medium show good agreement in case highly enriched uranium / water lattices. The Disadvantage factor and two neutron fluxes in case uranium oxide / water lattices also are calculated.

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REFERENCES

(1) M.M. Nanneh and M.M.R. Williams; Ann Nuclm Eng; 6, 337 (1986).
(11) A. Elghazaly; Arab J. of Nucl Scie and Applications; 43(4), 278 (2010).