On Selection of the Probability Distribution for Representing the Maximum Annual Wind Speed in East Cairo, Egypt

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ABSTRACT

The main objective of this paper is to identify an appropriate probability model and best plotting position formula which represent the maximum annual wind speed in east Cairo. This model can be used to estimate the extreme wind speed and return period at a particular site as well as to determine the radioactive release distribution in case of accident occurrence at a nuclear power plant. Wind speed probabilities can be estimated by using probability distributions. An accurate determination of probability distribution for maximum wind speed data is very important in expecting the extreme value. The probability plots of the maximum annual wind speed (MAWS) in east Cairo are fitted to six major statistical distributions namely: Gumbel, Weibull, Normal, Log-Normal, Logistic and Log-Logistic distribution, while eight plotting positions of Hosking and Wallis, Hazen, Gringorten, Cunnane, Blom, Filliben, Benard and Weibull are used for determining exceedance of their probabilities. A proper probability distribution for representing the MAWS is selected by the statistical test criteria in frequency analysis. Therefore, the best plotting position formula which can be used to select appropriate probability model representing the MAWS data must be determined. The statistical test criteria which represented in: the probability plot correlation coefficient (PPCC), the root mean square error (RMSE), the relative root mean square error (RRMSE) and the maximum absolute error (MAE) are used to select the appropriate probability position and distribution. The data obtained show that the maximum annual wind speed in east Cairo vary from 44.3 Km/h to 96.1 Km/h within duration of 39 years. Weibull plotting position combined with Normal distribution gave the highest fit, most reliable, accurate predictions and determination of the wind speed in the study area having the highest value of PPCC and lowest values of RMSE, RRMSE and MAE.

Keywords: Maximum Annual Wind Speed / Probability Positions / Probability Distributions / Goodness of Fit Tests

INTRODUCTION

The potential danger from an accident at a nuclear power plant is exposure to radiation. This exposure could come from the release of radioactive material from the plant into the environment. The area which may be affected by the radioactive material is determined by the amount released from the plant, wind direction and speed, and weather conditions. The major hazards to people in the vicinity are radiation exposure to the body and inhalation and ingestion of radioactive materials. The main objective purpose of this paper is to identify an appropriate probability model which represents the maximum annual wind speed in east Cairo area. This model can be used in estimating the extreme wind speed. The data of the maximum annual wind speed (MAWS) in the east Cairo at latitude 30.1114^{o} N and longitude 31.4139^{o} E is collected[(1)]. The available wind speed data of the east Cairo of Egypt is obtained for year 1973 to 2011. However, several frequency of two parameter distribution functions have been proposed in literature and used to model wind speed, such as the Gumbel distribution[2-6], the Weibull distribution[5-9], the Log-Normal distribution[9,10] and the Normal distribution[11], the Logistic distribution[12] and the Log-Logistic distribution[10,13]. Moreover, There are several plotting position formulas are used for estimating the exceedance probability function such as Weibull[2,14,15,1616,21], Gringorten[17-21], Hazen[18,21], Cunnane[18-20], Blom[18-21], Benard[18,21], Filliben[22] as well as Hosking and Wallis[23]. The study is based on selecting the best probability plotting position and an appropriate probability distribution representing the data. Eight formulas of probability position namely: Hosking and Wallis[24], Hazen[25], Gringorten[26], Cunnane[27], Blom[28], Filliben[29], Benard[30] and Weibull[31] are compared to choose the best one that can be used for constructing some distribution probability plots. Six theoretical probability distributions are compared to present the more appropriate model for representing the MAWS in east Cairo. More specifically, the six probability models which have two parameters, are the Gumbel (extreme value type I (EVI)), the Weibull, the Normal, the Log-Normal, the Logistic and the Log-Logistic distribution. The performance of different probability positions and distributions is compared using four statistical test criteria. Many researchers used the goodness of fit test criteria for measuring the performance of statistical probability distributions such as root mean square error (RMSE)[18,32], relative root mean square error (RRMSE)[20,32], maximum absolute error (MAE)[32] probability plot correlation coefficient (PPCC)[20, 32, 33, 34] and coefficient of determination (R^2)[18, 33].

In this work, the probability plot correlation coefficient (PPCC), the root mean square error (RMSE), the relative root mean square error (RRMSE) and the maximum absolute error (MAE) are calculated. The results show that, the Weibull plotting position combined with the Normal distribution gave the highest fit, most reliable and accurate predictions of the wind speed in the study area having the highest value of PPCC and lowest values of RMSE, RRMSE and MAE.

METHODOLOGY

1. Probability distributions

Several probability models have been developed to describe the distribution of annual extreme wind speed at a single site. However, the choice of a suitable probability model is still one of the major problems in engineering practice. The selection of an appropriate probability model depends mainly on the characteristics of available wind speed data at the particular site. Hence, it is necessary to evaluate many available distributions in order to find a suitable model that could provide accurate extreme wind speed estimates. Therefore, the main objective of the present study is to propose a general procedure for evaluating systematically the performance of various distributions. Six probability distributions are investigated, namely: Gumbel (extreme value type I (EVI)), Weibull, Normal, Log-Normal, Logistic and Log-Logistic distribution. The probability density function (PDF) $f(x)$ and cumulative distribution function (CDF) $F(x)$ and the associated parameters for each of probability distribution are:

Gumbel (EVI) distribution:

$$f(x) = (1/s)\exp\{-(x-m)/s - \exp[-(x-m)/s]\}, \quad -\infty \prec x \prec \infty \tag{1}$$

$$F(x) = \exp\{-\exp[-(x-m)/s]\} \tag{2}$$

$f(x)$ is the PDF, $F(x)$ is the CDF, m is the location parameter and s is the scale paeameter .

Weibull distribution:

$$f(x) = s^{-l}\, l\, x^{l-1} \exp\{-(x/s)^{l}\}, \quad 0 \le x,\ 0 \prec s, l \tag{3}$$

$$F(x) = 1 - \exp\{-(x/s)^{l}\} \tag{4}$$

f(x) is the PDF, *F(x)* is the CDF, s is the scale parameter and l is the shape parameter .

Normal distribution:

$$f(x) = \frac{1}{s\sqrt{2p}} \exp\{-\tfrac{1}{2}[(x-m)/s\,]^2\}, \quad -\infty \prec x \prec \infty \tag{5}$$

$$F(x) = \Phi[(x-m)/s\,] \tag{6}$$

standardized from: m=0, s =1 where, Φ is CDF of standard Normal (0,1) distribution .

μ is the location parameter and σ is the scale parameter.

Log-Normal distribution:

$$f(x) = \frac{1}{xs_{\ln x}\sqrt{2p}} \exp\{-\tfrac{1}{2}[(\ln x - m_{\ln x})/s_{\ln x}]^2\}, \quad 0 \prec x \tag{7}$$

$$F(x) = \Phi[(\ln x - m_{\ln x})/s_{\ln x}] \tag{8}$$

standardized from: m=0, s =1 where, Φ is CDF of standard Normal (0,1) distribution .

μ is the scale parameter and σ is the shape parameter.

Logistic distribution:

$$f(x) = \frac{\exp\{(x-m)/s\,\}}{s\,[1+\exp\{(x-m)/s\,\}]^2}, \qquad -\infty \prec x \prec \infty \tag{9}$$

$$F(x) = \frac{\exp\{(x-m)/s\,\}}{1+\exp\{(x-m)/s\,\}} \tag{10}$$

μ is the location parameter and σ is the scale parameter.

Log-Logistic distribution:

$$f(x) = \frac{\exp\{(\ln x - m)/s\,\}}{s\,x[1+\exp\{(\ln x - m)/s\,\}]^2}, \qquad 0 \prec x \prec \infty \tag{11}$$

$$F(x) = \frac{\exp\{(\ln x - m)/s\,\}}{1+\exp\{(\ln x - m)/s\,\}} \tag{12}$$

μ is the scale parameter and σ is the shape parameter.

2. Plotting positions

Many empirical formulas have been proposed for the determination of plotting positions. Most of them can be expressed generally in the following form:

$$\hat{F}\ (x^{3}\ x_T) = (i-a)/(n+1-2a) \tag{13}$$

or in a general form:

$$\hat{F}\ (x^{3}\ x_T) = (i-a)/(n+b) \tag{14}$$

where: $\hat{F}$ is the exceedance probability of extreme value,
x_T is the extreme probable value,
a is a constant.

The existing plotting position formula in form of Eq. (13): a is equal to 0.5 for Hazen's formula; zero for Weibull's formula; 3/8 for Blom's formula; 0.44 for Gringorten's formula; 0.3 for Benard's formula; 0.4 for Cunnane's formula and 0.3175 for Filliben's formula.

The existing plotting position formula in form of Eq. (14): a is equal to 0.35 and b is equal to 0.0 for Hosking and Wallis. These plotting position formulas are listed in Table 1. The common technique of these formulas is to arrange the observed data in descending order of magnitude and assign an order number (i) to the rank value. Then the probability ($\hat{F}$) of each event being exceeded is determined using plotting position formula(21).

Table (1): plotting position formulas

Name	Source	Relationship (F_i)
Hosking and Wallis	Hosking and Wallis	*(i-0.35)/n*
Hazen	Hazen	*(i-0.5)/n*
Gringorten	Gringorten	*(i-0.44)/(n+0.12)*
Cunnane	Cunnane	*(i-0.4)/(n+0.2)*
Blom	Blom	*(i-0.375)/(n+0.25)*
Filliben	Filliben	*(i-0.3175)/(n+0.365)*
Benard	Benard	*(i-0.3)/(n+0.4)*
Weibull	Weibull	*i/(n+1)*

3. Probability plot

The probability plot is a graphical technique for assessing whether or not a data set follows a given distribution. The data are plotted against a theoretical distribution in such a way that the points should form approximately a straight line. The estimates of the two parameters of distributions can be found graphically using the probability plot. The correlation coefficient (PPCC) associated with the linear fit to the data in the probability plot is a measure of the goodness of the fit. Estimates of the location and scale parameters of the distribution are given by the intercept and slope. Probability plots can be generated for several competing distributions to see which provides the best fit, and the probability plot generating the highest correlation coefficient is the best choice since it generates the straightest probability plot.

To construct the probability plot the observed data x_i are ranked in ascending order, and denoted from $x_{1:n}$ to $x_{n:n}$, where n is the total number of observations. A plotting position of the non-exceedance probability $\hat{F}_{i:n}$ is computed for each $x_{i:n}$ using some formulas of plotting positions. Parameters for the compared distributions are computed using the regression scheme. The regression (graphical) method starts with the data in levels, x_i or logs, $ln x_i$. Let $\hat{F}_i$ is probability position. The horizontal axis $(g(\hat{F}_i))$ is a transformation of $\hat{F}_i$ (the reduce variate). The slope and intercept (denoted a and b), in the relationship between $g(\hat{F}_i)$ and x_i (in levels or logs), correspond to the parameters of the model. The quantiles function of the distributions is given by:

$$x_p = a\, g(\hat{F}_i) + b \qquad (15)$$

$$Ln x_p = a\, g(\hat{F}_i) + b \qquad (16)$$

where x_p (expected wind speed) is the quantiles function of the distributions.

The correlation coefficient (PPCC) measures the strength of a linear relationship between two variables. Hence the correlation of x_{pi} and $g(\hat{F}_i)$ might be a good way to measure how well each distribution fits the data. Table 2 shows the coordinates of X-axis and Y-axis of linear relationships (probability plots) of the compared probability distributions.

Table (2): the coordinates of X-axis and Y-axis of distributions probability plots

Model	Equation of Cumulative distribution function (CDF)	X-axis Reduce variate ($g(\hat{F}_i)$)	Y- axis
Gumbel (EVI)	$F(x) = \exp\{-\exp[-(x-m)/s]\}$	$-ln[-ln(\hat{F}_i)]$	x_i values
Weibull	$F(x) = 1-\exp\{-(x/s)^l\}$	$ln[-ln(1-\hat{F}_i)]$	lnx_i values
Normal	$F(x) = \Phi[(x-m)/s]$	$F^{-1}(\hat{F}_i)$	x_i values
Log-Normal	$F(x) = \Phi[(\ln x-m)/s]$	$F^{-1}(\hat{F}_i)$	lnx_i values
Logistic	$F(x) = \exp\{(x-m)/s\} \big/ 1+\exp\{(x-m)/s\}$	$ln[F_i/(1-\hat{F}_i)]$	x_i values
Log-Logistic	$F(x) = \exp\{(\ln x-m)/s\} \big/ 1+\exp\{(\ln x-m)/s\}$	$ln[F_i/(1-\hat{F}_i)]$	lnx_i values

Where, $F^{-1}(\ )$ denotes the inverse standard Normal cumulative distribution function.

4. Goodness of fit tests

It is very important to select an appropriate probability distribution in frequency analysis for the estimation of design quantile. An appropriate probability distribution and positions is selected generally based on the goodness of fit tests which is the method for examining the fitness between sample data and its population for a given probability distribution. For ease of computation, four test criteria are used.

The root mean square error (RMSE) also known as the standard error is the sum of squares of the differences between observed and computed values:

$$RMSE = [(x_i - x_{pi})^2 /(n - m)]^{1/2} \quad (17)$$

Where: x_i, i =1,…,n are the observed values,
x_{pi}, i = 1,…,n are the expected values computed from an assumed empirical probability distribution using plotting position formula based on the sorted ranks of observed values and the number of parameters estimated for the assumed distribution, denoted as *m*.

The relative root mean square error (RRMSE) is defined as:

$$RRMSE = \{S[(x_i - x_{pi}) / x_i]^2 /(n - m)\}^{1/2} \quad (18)$$

The maximum absolute error (MAE) is closely related to the Kolmogorov-Smirnov statistics and represents the largest absolute difference between the observed and computed values:

$$MAE = max(|x_i - x_{pi}|) \quad (19)$$

Probability plot correlation coefficient (PPCC)

PPCC associated with the linear fit to the data in the probability plot. The adequacy of a fitted probability position and distribution can be evaluated by the PPCC which is essentially a measure of the linearity of the probability plot(29). The probability plot correlation coefficient (PPCC) test has been known as powerful and easy test among the goodness of fit tests. The test uses the correlation between the ordered observations and the corresponding fitted quantilies, determined by plotting position $\hat{F}_i$ for each x_i.

The PPCC of the fitted distribution at is given by:

$$PPCC = S[(x_i - \bar{x})(x_{pi} - \bar{x}_{pi})] / [S(x_i - \bar{x})^2 \, S(x_{pi} - \bar{x}_{pi})^2]^{1/2} \quad (20)$$

Where $\bar{x}$ and $\bar{x}_{pi}$ denote the average value of the observations and fitted quantiles, respectively(32).

RESULTS AND DISCUSSIONS

This study is carried out on annual extreme wind speed of Cairo airport from year 1973 to 2011 which is collected from east Cairo. The largest values of annual wind speeds recorded in east Cairo are shown in time series given in Figure (1). The highest wind speed of 96.1Km/h was observed in 1985 and declined to 44.3Km/h in the year 2002.

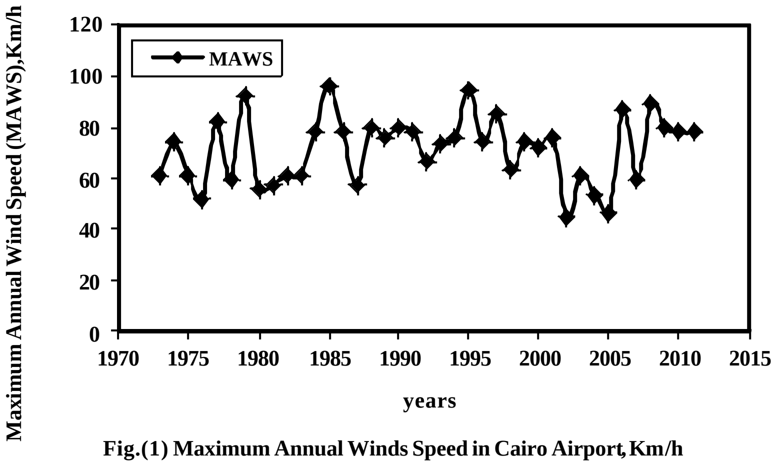


Fig.(1) Maximum Annual Winds Speed in Cairo Airport, Km/h

The maximum annual wind speeds are fitted to the Gumbel, Weibull, Normal, Log-Normal, Logistic and Log-Logistic distributions and the non-exceedance probabilities determined using the various plotting positions formula in Table 2. The determined probabilities of non-exceedance of the MAWS *($\hat{F}_i$)* before subjecting them to the statistical distributions are shown in Table 3.

Table (3): the probability ($\hat{F}_i$) of each event being not exceeded is determined using plotting position formulas

years	MAWS Km/h	Hosking and Wallis	Hazen	Gringorten	Cunnane	Blom	Filliben	Benard	Weibull
2002	44.3	0.016667	0.012821	0.014315	0.015306	0.015924	0.017338	0.017766	0.025
2005	46.5	0.042308	0.038462	0.039877	0.040816	0.041401	0.042741	0.043147	0.05
1976	51.9	0.067949	0.064103	0.06544	0.066327	0.066879	0.068144	0.068528	0.075
2004	53.5	0.09359	0.089744	0.091002	0.091837	0.092357	0.093548	0.093909	0.1
1980	55.4	0.119231	0.115385	0.116564	0.117347	0.117834	0.118951	0.119289	0.125
1981	57.2	0.144872	0.141026	0.142127	0.142857	0.143312	0.144354	0.14467	0.15
1987	57.2	0.170513	0.166667	0.167689	0.168367	0.16879	0.169757	0.170051	0.175
1978	59.1	0.196154	0.192308	0.193252	0.193878	0.194268	0.195161	0.195431	0.2
2007	59.4	0.221795	0.217949	0.218814	0.219388	0.219745	0.220564	0.220812	0.225
1983	60.7	0.247436	0.24359	0.244376	0.244898	0.245223	0.245967	0.246193	0.25
1973	61.1	0.273077	0.269231	0.269939	0.270408	0.270701	0.271371	0.271574	0.275
1975	61.1	0.298718	0.294872	0.295501	0.295918	0.296178	0.296774	0.296954	0.3
1982	61.1	0.324359	0.320513	0.321063	0.321429	0.321656	0.322177	0.322335	0.325
2003	61.1	0.35	0.346154	0.346626	0.346939	0.347134	0.34758	0.347716	0.35
1998	63	0.375641	0.371795	0.372188	0.372449	0.372611	0.372984	0.373096	0.375
1992	66.5	0.401282	0.397436	0.397751	0.397959	0.398089	0.398387	0.398477	0.4
2000	72	0.426923	0.423077	0.423313	0.423469	0.423567	0.42379	0.423858	0.425
1993	73.7	0.452564	0.448718	0.448875	0.44898	0.449045	0.449193	0.449239	0.45
1974	74.1	0.478205	0.474359	0.474438	0.47449	0.474522	0.474597	0.474619	0.475
1996	74.1	0.503846	0.5	0.5	0.5	0.5	0.5	0.5	0.5
1999	74.1	0.529487	0.525641	0.525562	0.52551	0.525478	0.525403	0.525381	0.525
1989	75.9	0.555128	0.551282	0.551125	0.55102	0.550955	0.550807	0.550761	0.55
1994	75.9	0.580769	0.576923	0.576687	0.576531	0.576433	0.57621	0.576142	0.575
2001	75.9	0.60641	0.602564	0.602249	0.602041	0.601911	0.601613	0.601523	0.6
1984	77.8	0.632051	0.628205	0.627812	0.627551	0.627389	0.627016	0.626904	0.625
1986	77.8	0.657692	0.653846	0.653374	0.653061	0.652866	0.65242	0.652284	0.65
1991	77.8	0.683333	0.679487	0.678937	0.678571	0.678344	0.677823	0.677665	0.675
2010	77.8	0.708974	0.705128	0.704499	0.704082	0.703822	0.703226	0.703046	0.7
2011	77.8	0.734615	0.730769	0.730061	0.729592	0.729299	0.728629	0.728426	0.725
1988	79.5	0.760256	0.75641	0.755624	0.755102	0.754777	0.754033	0.753807	0.75
1990	79.5	0.785897	0.782051	0.781186	0.780612	0.780255	0.779436	0.779188	0.775
2009	79.5	0.811538	0.807692	0.806748	0.806122	0.805732	0.804839	0.804569	0.8
1977	81.7	0.837179	0.833333	0.832311	0.831633	0.83121	0.830243	0.829949	0.825
1997	84.8	0.862821	0.858974	0.857873	0.857143	0.856688	0.855646	0.85533	0.85
2006	87	0.888462	0.884615	0.883436	0.882653	0.882166	0.881049	0.880711	0.875
2008	88.9	0.914103	0.910256	0.908998	0.908163	0.907643	0.906452	0.906091	0.9
1979	92.4	0.939744	0.935897	0.93456	0.933673	0.933121	0.931856	0.931472	0.925
1995	94.3	0.965385	0.961538	0.960123	0.959184	0.958599	0.957259	0.956853	0.95
1985	96.1	0.991026	0.987179	0.985685	0.984694	0.984076	0.982662	0.982234	0.975

Probability plot is a graphical technique for assessing whether or not a data set follows a given distribution. The data are ranked according to 8 probability positions. The ranked data are evaluated with 6 methods of probability distribution functions. Probability plot is constructed according to Table 2 by using Table 3.

To construct a probability plot, the sample order statistic x_i or ln x_i is plotted (usually on the vertical axis) against $g(\hat{F}_i)$ (usually on the horizontal axis). The mathematical expressions obtained for various probability distributions at different formulas of plotting position are presented in Table 4. The mathematical expressions represent the expected extreme wind speed in the east Cairo. The equations are obtained from 48 constructing probability plots.

Table (4): mathematical expressions (predicted values) for various probability distributions at different formulas of probability position

Probability Position	**Gumbel**	**Weibull**	**Normal**	**Log-Normal**	**Logistic**	**Log-Logistic**
Hosking and Wallis	$x_p = 9.5674g(\hat{F}_i)+65.224$	$Ln(x_p)= 0.1532g(\hat{F}_i)+4.3277$	$x_p = 12.909\,g(\hat{F}_i)+70.69$	$Ln(x_p)= 0.1876g(\hat{F}_i)+ 4.2407$	$x_p = 7.169\,g(\hat{F}_i)+70.648$	$Ln(x_p)= 0.1042\,g(\hat{F}_i)+ 4.2401$
Hazen	$x_p = 9.9286g(\hat{F}_i)+65.304$	$Ln(x_p) = 0.1496g(\hat{F}_i)+4.3299$	$x_p = 12.953\,g(\hat{F}_i)+70.96$	$Ln(x_p)= 0.1887g(\hat{F}_i)+4.2446$	$x_p = 7.2155\,g(\hat{F}_i)+70.96$	$Ln(x_p)= 0.1052\,g(\hat{F}_i)+4.2446$
Gringorten	$x_p = 10.074\,(\hat{F}_i)+65.261$	$Ln(x_p)= 0.1517g(\hat{F}_i)+4.3305$	$x_p = 13.082\,g(\hat{F}_i)+70.96$	$Ln(x_p)= 0.1906g\,(\hat{F}_i)+4.2446$	$x_p = 7.3185\,g(\hat{F}_i)+70.96$	$Ln(x_p)= 0.1067\,g(\hat{F}_i)+ 4.2446$
Cunnane	$x_p = 10.166g(\hat{F}_i)+65.235$	$Ln(x_p)= 0.153g(\hat{F}_i)+ 4.3308$	$x_p = 13.165\,g(\hat{F}_i)+70.96$	$Ln(x_p)= 0.1918g(\hat{F}_i)+ 4.2446$	$x_p = 7.3836\,g(\hat{F}_i)+70.96$	$Ln(x_p)= 0.1077\,g(\hat{F}_i)+ 4.2446$
Blom	$x_p = 10.221g(\hat{F}_i)+65.219$	$Ln(x_p)= 0.1538g(\hat{F}_i)+4.3311$	$x_p = 13.215\,g(\hat{F}_i)+70.96$	$Ln(x_p)= 0.1925g(\hat{F}_i)+ 4.2446$	$x_p = 7.423\,g(\hat{F}_i)+70.962$	$Ln(x_p)= 0.1082\,g(\hat{F}_i)+4.2446$
Filliben	$x_p = 10.344g(\hat{F}_i)+65.185$	$Ln(x_p)= 0.1556g(\hat{F}_i)+4.3315$	$x_p = 13.327\,g(\hat{F}_i)+70.96$	$Ln(x_p)= 0.1941g(\hat{F}_i)+4.2446$	$x_p = 7.5103g(\hat{F}_i)+70.96$	$Ln(x_p)= 0.1095\,g(\hat{F}_i)+ 4.2446$
Benard	$x_p = 10.38g(\hat{F}_i)+65.174$	$Ln(x_p)= 0.1561g(\hat{F}_i)+4.3317$	$x_p = 13.36\,g(\hat{F}_i)+70.96$	$Ln(x_p)= 0.1946g(\hat{F}_i)+ 4.2446$	$x_p = 7.5361\,g(\hat{F}_i)+70.96$	$Ln(x_p)= 0.1099\,g(\hat{F}_i)+ 4.2446$
Weibull	$x_p = 10.939g(\hat{F}_i)+65.021$	$Ln(x_p)= 0.1641g(\hat{F}_i)+4.3338$	$x_p = 13.887\,g(\hat{F}_i)+70.96$	$Ln(x_p)= 0.2022g(\hat{F}_i)+ 4.2446$	$x_p = 7.934\,g(\hat{F}_i)+70.962$	$Ln(x_p)= 0.1156\,g(\hat{F}_i)+ 4.2446$

Selection of Probability Distributions and plotting Positions

The selection of an appropriate plotting plot formula for each statistical distribution is the most important step that is generally chosen by using the goodness of fit tests. The PPCC test is a goodness-of-fit test which measures and evaluates the linearity of the probability plot.

Comparison between eight formulas of plotting position for six probability distributions is done to choose the best plotting position and the appropriate probability distribution. The comparison is based on four test criteria namely: probability plot correlation coefficient (PPCC), root mean square error (RMSE), relative root mean square error (RRMSE) and maximum absolute error (MAE). Table 5 shows the PPCC, RMSE, RRMSE and MAE between observed and expected values for all compared distributions using different formula of plotting positions. The best distribution and plotting position is determined according to the highest PPCC and the lowest RRMSE and MAE between observed and expected values. It is found that the Normal distribution has the maximum value of PPCC and the minimum values of RMSE, RRMSE and MAE of MAWS under each plotting position when it is compared with other distributions. Moreover, The Weibull plotting position has the highest value of PPCC and the lowest values of RMSE, RRMSE and MAE under each probability distribution when it is compared with other plotting positions.

The results show that the Weibull' plotting position at each distribution has slightly, highest value of PPCC and lowest value RMSE, RRMSE and MAE comparing with Benard and Filliben plotting position at the same distribution. Then, The Weibull plotting is considered the best formula for selection an appropriate distribution, followed by Benard and Filliben, respectively. However, the Hosking and Wallis plotting position at Weibull distribution is occupy third rank after Benard plotting position. The appropriate probability distribution which representing and excpecing the (MAWS) in east Cairo is Normal distribution followed by Weibull and Logistic distribution, respectively. The results of Table 5 indicate that the Weibull plotting position combined with Normal distribution give the highest fit, most reliable and accurate predictions of the MAWS east Cairo.

Table (5): The test criteria of different formulas of plotting position for the six compared distributions

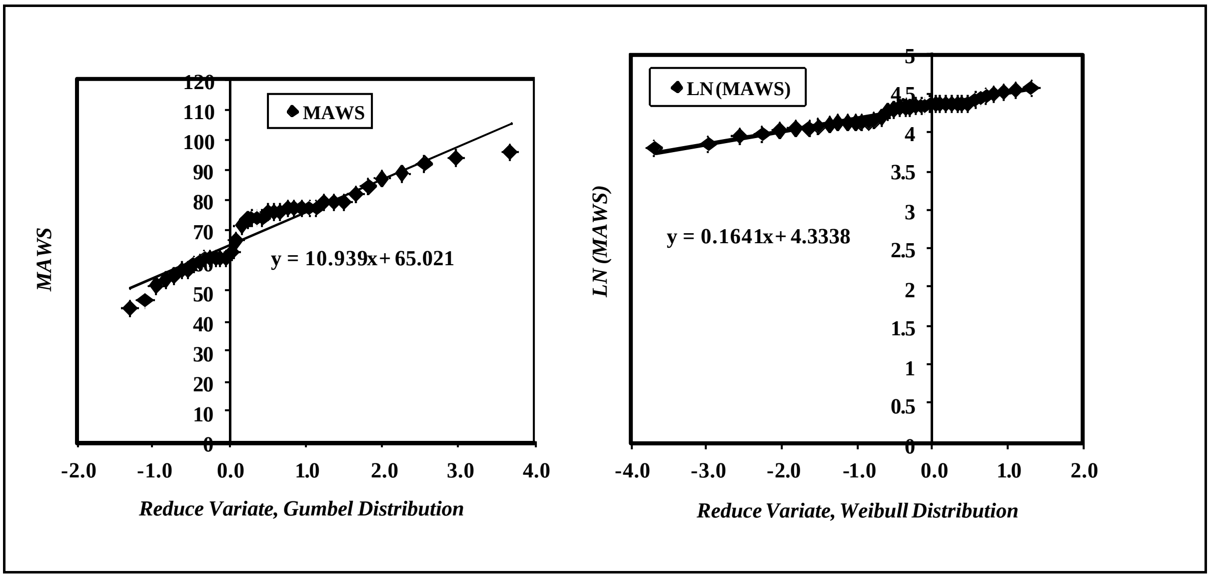
		PPCC	RMSE	RRMSE	MAE
Gumbel					
	Weibull	0.962428	3.60902	0.054804	9.135469
	Benard	0.957892	3.816265	0.057238	10.81708
	Filliben	0.957542	3.831737	0.057423	10.93825
	Blom	0.956298	3.886239	0.058073	11.35156
	Cunnane	0.955706	3.911886	0.05838	11.54552
	Gringorten	0.954684	3.955737	0.058909	11.86722
	Hazen	0.952946	4.029084	0.059803	12.39603
	Hosking and Wallis	0.944557	4.364136	0.065102	14.17574
Weibull					
	Weibull	0.984423	2.317373	0.03461	5.293756
	Benard	0.982095	2.424945	0.036981	5.517268
	Filliben	0.981898	2.433873	0.037166	5.527232
	Blom	0.981181	2.466554	0.037854	5.586213
	Cunnane	0.980831	2.482686	0.038175	5.604537
	Gringorten	0.980214	2.50961	0.038741	5.647132
	Hazen	0.979134	2.557409	0.039703	5.709897
	Hosking and Wallis	0.982013	2.414627	0.037034	5.498528

Normal					
	Weibull	0.985993	2.216801	0.031988	4.511053
	Benard	0.985354	2.266427	0.032875	4.631627
	Filliben	0.985294	2.271039	0.032958	4.639662
	Blom	0.98507	2.288139	0.033268	4.667573
	Cunnane	0.984958	2.296624	0.033422	4.680275
	Gringorten	0.984758	2.311737	0.033697	4.701851
	Hazen	0.984399	2.338602	0.034187	4.736169
	Hosking and Wallis	0.983493	2.404982	0.034163	5.141392
LogNormal					
	Weibull	0.978748	2.89681	0.039802	7.538043
	Benard	0.978418	2.980874	0.040164	8.872035
	Filliben	0.97838	2.987394	0.040205	8.963377
	Blom	0.978232	3.012716	0.040364	9.300471
	Cunnane	0.978156	3.025267	0.040447	9.460694
	Gringorten	0.978015	3.046048	0.040597	9.719689
	Hazen	0.977754	3.081486	0.040874	10.14033
	Hosking and Wallis	0.974624	3.341523	0.043845	12.17648
Logistic					
	Weibull	0.983891	2.376082	0.035028	4.950543
	Benard	0.981308	2.557818	0.03816	5.100711
	Filliben	0.981113	2.570999	0.038394	5.183935
	Blom	0.980404	2.618351	0.039236	5.473735
	Cunnane	0.980058	2.641093	0.03964	5.607886
	Gringorten	0.979451	2.680602	0.040341	5.834141
	Hazen	0.978389	2.748265	0.041538	6.204728
	Hosking and Wallis	0.976521	2.863207	0.0407	8.273621
Log-Logistic					
	Weibull	0.977122	3.145717	0.041462	10.39603
	Benard	0.975251	3.34553	0.043335	12.27261
	Filliben	0.975088	3.360003	0.043492	12.39381
	Blom	0.974492	3.41391	0.044068	12.84099
	Cunnane	0.974199	3.443002	0.044352	13.088
	Gringorten	0.97368	3.486268	0.044841	13.42557
	Hazen	0.972763	3.563297	0.045696	14.01974
	Hosking and Wallis	0.967627	4.007631	0.050209	17.22902

Conseqently, the mathematical expression which can be used for expecting the extreme wind speed in east Cairo is:

$$x_p = 13.887\,g(\hat{F}_i)+70.96$$

Graphical techniques can be used to visually assess the adequacy of a fitted distribution. The probability plots of the six compared distributions using Weibull probability position are shown in Figure 2. The Q-Q plots of the compared distributions using Weibull probability position are shown in Figure 3. From the Q-Q plot it is obvious that the Normal distribution presenting the best fit for MAWS data followed by the Weibull distribution.



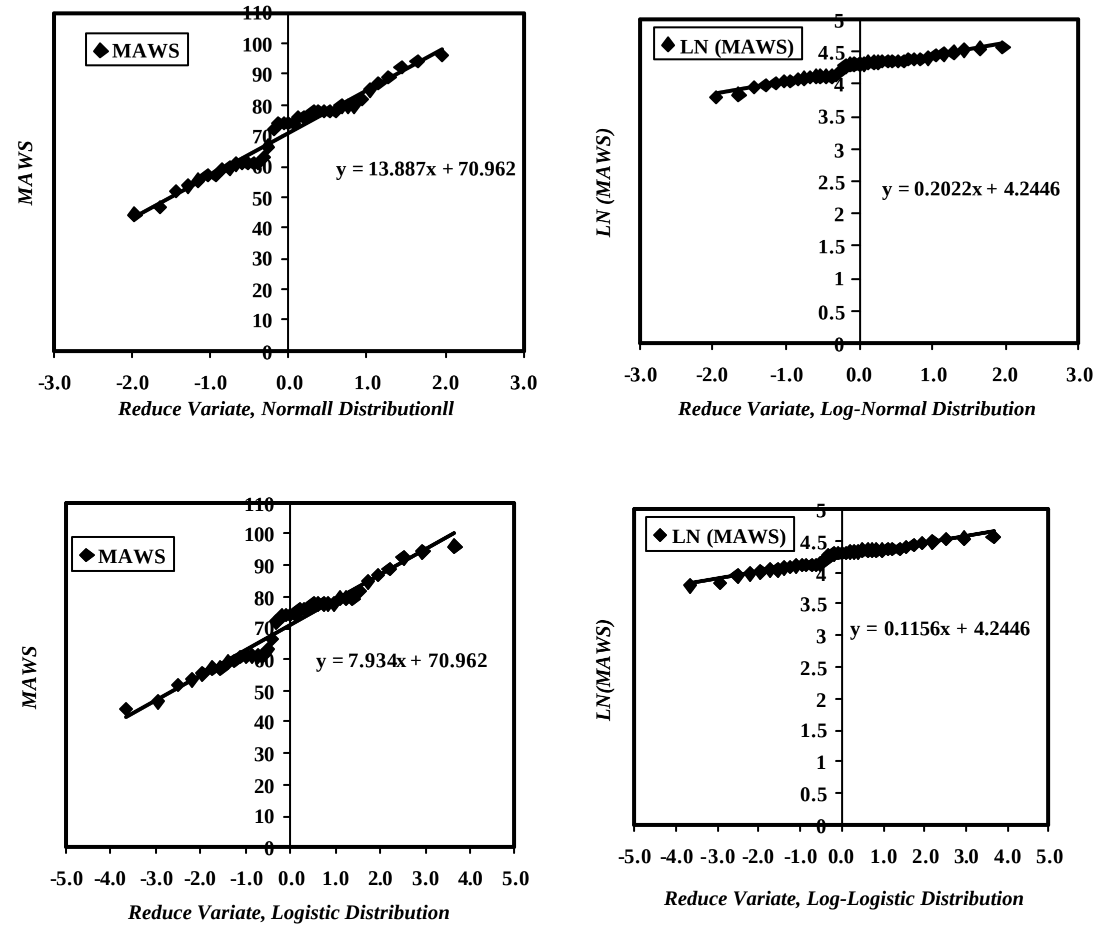
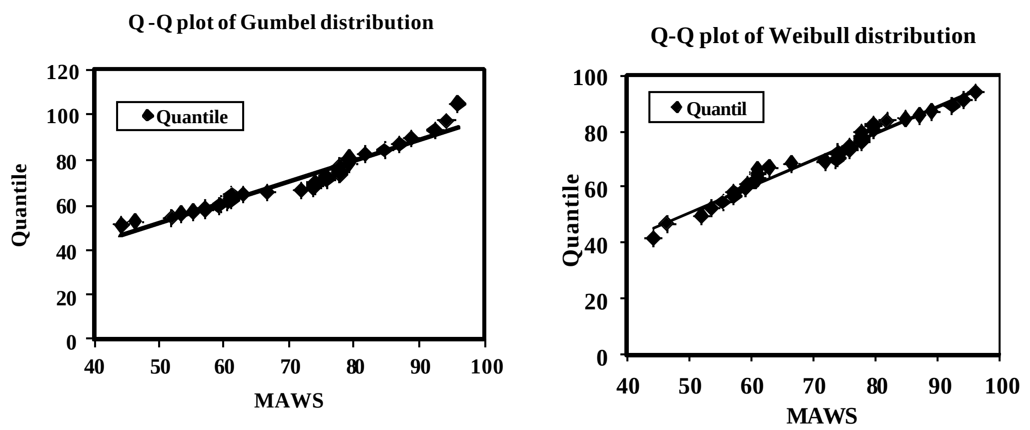


Figure (2) The probability plots of compared distributions using Weibull probability position



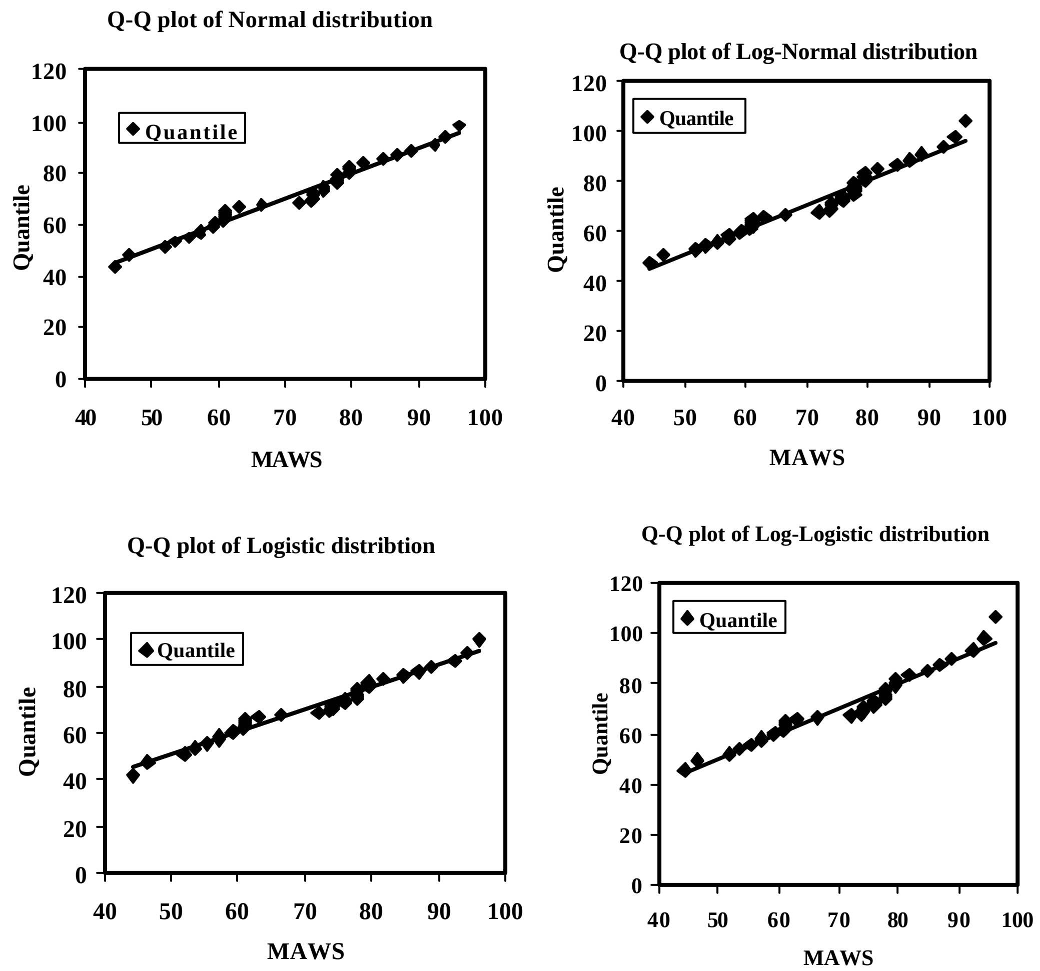


Figure (3) The Q-Q plots of compared distributions using Weibull probability position

CONCLUSION

In this study, the maximum annual wind speed (MAWS) is plotted against their hydrologic years. Six probability distributions and eight plotting positions are compared. Fourty eight probability plots are constructed to represent the expected extreme wind speed and choose the best distribution represents the MAWS in the east Cairo. The performances of the probability distributions are assessed using the probability plot correlation coefficient (PPCC), the root mean square error (RMSE), the relative root mean square error (RRMSE) and maximum absolute error (MAE). The results proved that the Weibull plotting position is considered the best probability position which can be used for selecting the appropriate distribution for representing and expecting the MAWS in the east Cairo. The best probability positions which followed Weibull plotting position are Benard and Filliben, respectively. The appropriate probability distribution model which representing and expecting the MAWS in east Cairo is the Normal distribution followed by Weibull and Logistic distribution, respectively. Then, Weibull plotting position combined with Normal distribution gave the highest fit, most reliable and accurate predictions of the wind speed in the study area.

REFERENCES

1) World Weather-TUTiempo, www.tutiempo.net/en.
2) Brendan Murphy and Peter L. Jackson; Extreme Value Analysis: Return Periods of Severe Wind Events in the Central Interior of British Columbia, Report Prepared for McGregor Model Forest Association, (1997).
3) Lasse Makkonen; Problems in the extreme value analysis; Structural Safety Journal; 30, 5, 405-419 (2008).
4) Melissa D. Burton, Andrew C. Allsop; Predicting Design Wind Speeds from Anemometer Records: Some Interesting Findings, 11th American Conference on Wind Engineering-San JuaN,Puerto Rico,; 22-26, (2009).
5) Xiao, Y. Q., Li, Q. S., Li, Z. N., Chow, Y.W., and Li, G.Q.; Probability Distribution of Extreme Wind Speed and its Occurrence Interval; Engineering Structures Joural; 28, 1173-1181 (2006).
6) A.M. Dubi; Frequency and long-term distribution of coastal winds of Tanzania, gridnairobi. unep.org/.../Tanzania/Coastal_winds
7) Paritosh Bhattacharya and Rakhi Bhattacharjee; A Study on Weibull Distribution for Estimating the Parameters; Journal of Applied Quantitative Methods, Vol. 5, Issue 2, Pages 234-241, (2010).
8) Ahmad Mahir Razali, Rozaimah Zainal Abidin, Azami Zaharim and Kamaruzzaman Sopian; Fitting of Statistical Distributions to Wind Speed Data in Malaysia; European Journal of Scientific Research; Vol. 26, No. 1, Pages 6-12, (2009).
9) Atul Viraj Wadagale, P.V. Thatkar, R.K. Dase and D.V. Tandale; Modified Anderson Darling Test for Wind Speed Data; International Journal of Computer Science & Emerging Technologies, Vol. 2, Issue 2, pages 249-251, (2011).
10) Veysel Yilmaz and H. Eray Çelik; A Statistical Approach to Estimate the Wind Speed Distribution: The Case of Gelibolu Region; l Dogus Üniversitesi Dergisi, Vol. 9, pages 122-132, (2008).
11) Kyungnam Ko, Kyoungbo Kim and Jongchul Huh; Variations of Wind Speed in Time on Jeju Island, Korea; Energy Journal, Vol. 35, pages 3381-3387, (2010).
12) Rodney Coleman; Modelling Extremes; Statistical Research Center for Complex Systems International Statistical Workshop, Seoul National University, (2002).
13) Chiodo, E. Lauria, D.; Analytical Study of Different Probability Distributions for Wind Speed Related to Power Statistics; International Conference on Clean Electrical Power, pages 733–738, (2009).
14) Lasse Makkonen; Notes and Correspondence Plotting Positions in Extreme Value Analysis; Journal of QApplied Mmeteorology and Climatology, Vol. 45, Pages 334-340, (2005).
15) F. Ola. Okulaja; The Frequency Distribution of Lagos / Ikeja Wind Gusts; Journals .ametsoc .org/, pages 379-383, (1967).
16) Diana S . Sinton and Julia A. Jones; ExtremeWinds and W indthrow in the Western Columbia River Gorge; Northwest Science Assocation, Vol. 76, No. 2, pages 173-182, (2002).
17) J. D. Holmes; Wind loading and structural response, Lecture 4; Prediction of Design Wind Speeds, www.hurricaneengineering.lsu.edu/.../03Lect4Desi.

18) Omotayo B. Adeboye and Michael O. Alatise; Performance of Probability Distributions and Plotting Positions in Estimating the Flood of River Osun at Apoje Sub-basin, Nigeria; Agricultural Engineering International: the CIGR Ejournal Manuscript LW 07 007. Vol. IX, (2007).
19) Sooyoung Kim and Jun-Haeng Heo; Comparison of the Probability Plot Correlation Coefficient Test Statistics for the General Extreme Value Distribution; World Environmental and Water Resources Congress, Pages 2456-2466, (2010).
20) Md. Abdul Karim and Jafflr Uddin Chowdhury; A Comparison of Four Distributions Used in Food Frequency Analysis in Bangladesh; Hydrological Sciences-Journal-des Sciences Hydrologiques, Vol. 40, No. l, (1995).
21) Ali El-Naqa and Nasser Abu Zeid; A Program of Frequency Analysis Using Gumbel's Method; Ground Water, Vol. 31, No. 6, (1993).
22) Stephen W. Looney, Thomas R. Gulledge, Jr.; Use of the Correlation Coefficient with Normal Probability Plots; The American Statistician, Vol. 39, No.1, pages 75-79, (1985).
23) Hidalgo-Muñoz, J. M., D. Argüeso, D. Calandria-Hernández, S.R. Gámiz-Fortis, M. J. Esteban-Parra and Y. Castro-Díez; Extreme Value Analysis of Precipitation Series in the South of Iberian Peninsula; http://cccma.seos.uvic.ca/ETCCDMI/software.shtml.
24) Hosking, J. R. and J. R. Wallis; A comparison of unbiased and plotting-position estimators of L moments; Water Resour. Res., Vol. 31, Pages 2019–2025, (1995).
25) Hazen, A.;Flood Flows, Study of Frequency and Magnitudes; Wiley, New York, (1930).
26) Gringorten, I. I.; A plotting rule for extreme probability paper; Journal of Geophysical Research, Vol. 68, No. 3, Pages 813–814, (1963).
27) Cunnane, C.; Unbiased Plotting Positions; A Rev~ew., Journal of Hydrology, Vol. 37, Pages 205-222, (1978).
28) Blom, G.; Statistical Estimates and Transformed Beta Variables; New York, John Wiley, (1958).
29) Filliben, J. J.; The Probability Plot Correlation Coefficient Test for Normality, Technometrics Vol. 17, Pages 111- 117, (1975).
30) Benard, A., and Bos-Levenbach, E. C.; "The Plotting of Observations on Probability Paper, Sratisrica, Vol. 7, Pages 163- 173. (1953)
31) Weibull, W.; A statistical theory of strength of materials; Ing Vetensk Akad Handl.,151, 1-45, (1939).
32) D. Q. Tao, V. T. V. Nguyen, A. Bourque; On selection of Probability Distributions for Representing Extreme Precipitations in Southern Quebec; Annual Conference of the Canadian Society for Civil Engineering, (2002).
33) Olofintoye O.O., Sule B.F. and Salami A.W.; Best–Fit Probability Distribution Model for Peak Daily Rainfall of Selected Cities in Nigeria; New York Science Journal, Vol. 2, ISSN 1554-0200, (2009).
34) Richard M. Vogel; The Probability Plot Correlation Coefficient Test for the Normal, Log-Normal, and Gumbel Distributional Hypotheses; Water Resources Research, Vol. 22, No.4, (1986).