Arab J. Nucl. Sci. Appl., Vol. 51, 3, 104-112 (2018)

Arab Journal of Nuclear Sciences and Applications

ISSN 1110-0451 *Web site: ajnsa.journals.ekb.eg* (ESNSA)

Elastic Scattering Cross Section of Pion (π^+) with N, ^{16}O and ^{28}Si

Sh.M.E. Sewailem and K.M. Hanna

Mathematics and Theoretical Physics Department, Nuclear Research Center, Atomic Energy Authority, Cairo, Egypt

Received 12th Dec. 2017
Accepted 23rd April 2018

The pion–nucleon and nucleus scattering at intermediate energies have been intensively studied in recent years on the basis of meson exchange theory. In this work, the optical potentials of the pion-nucleon (π^+ *N*) interactions are obtained on the basis of the single-boson-exchange (SBE). Sets of boson parameters suggested by the Julich group were used to obtain the radial forms of the interacting pion nucleon optical potential V(r), and the effective nuclear density of the target nuclei (oxygen and Silicon) was utilized to obtain the pion nucleus optical potential. The differential cross sections for (π^+ ^{16}O) and (π^+ ^{28}Si) are calculated using the first born approximation, the scattering angle in the frame of the center of mass system, within the framework of time independent Schrodinger equation, and in the π^+ energy region P_{lab} < 0.5 GeV. All results are found to be in a good agreement with the available experimental data.

Keywords: Strong interaction; Meson exchange; effective nuclear density; pion -nucleus optical potentials

Introduction

The pion elastic scattering from nuclei is an important constituent in the understanding of pion nucleus interactions. Without an understanding of elastic scattering, it is not possible to investigate the physics of pion production, pion absorption and pion charge exchange. The pion-nucleon interactions are very strong, the pion mesons have zero spin and occur in three isospin states (π^+, π^-, π^0) and their quark structure are $u\bar{d}$, $d\bar{u}$ and ($u\bar{u}$ - $d\bar{d}$) respectively. It is well known that the quark structures for proton and neutron are (uud) and (ddu) respectively [1, 2], and two isotopic (1/2) forms for proton and neutron Fig (1-A, 1-B).

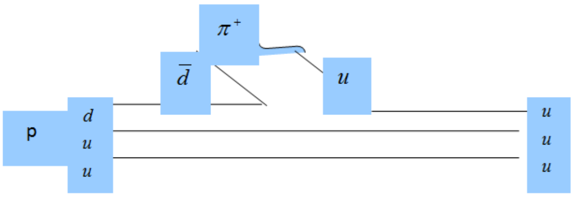


Fig. (1-A): Quark diagrams for (π^+ P)

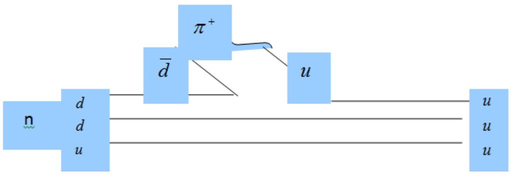


Fig. (1-B): Quark diagrams for (π^+ n)

Corresponding author: sh_m_sw@yahoo.com
DOI: 10.21608/ajnsa.2276.1020

The isospin couplings lead us to expect that this reaction is dominated by the π^+ proton interaction. Pion elastic scattering at 500 MeV is much less dominated by the nuclear surface, and is less sensitive to neutron and proton differences [3]. Many models for pion-nucleus scattering are built upon the impulse approximation, using a sum interaction of the mesons with free nucleons multiplied by the nuclear density [4].The pion nucleon scattering process is based on the exchange of two mesons, one is attractive scalar meson and the other is repulsive vector meson. Therefore, σ and ρ exchange, which are replaced by the correlated ($2\pi^+$) exchange potentials were used [5]. By this theory we construct an optical potential between the pion and nucleon, hence by using the density distribution and single folding method, the potential between the pion and nucleus have been considered [6].The magnitude of this potential reflects the scattering of the incident particle by the individual nucleons bound in the nucleus.

The mathematical formulation of the constructed optical potential is given in section II of this study. The effectiveness of this potential by presenting the angular distributions for elastic scattering of (π^+ ^{16}O) and (π^+ ^{28}Si) at energies of 114, 163 and 240 MeV, and at range of angle 10-60 degree is investigated in section III. The results are compared to the corresponding experimental data for pion-nucleus. Section IV provides π^+ nucleon and studied nuclei data conclusions.

The calculations of pion- nucleon and pion nucleus scattering show a good agreement with the experimental data and that the model presented here is successful for describing these types of nuclear interaction.

Theoretical Scheme

The pion nuclear interaction is described in frame of Klein Gordon or Schrodinger equation [7, 8] using the distorted wave function in case of Born approximation. The instructive approach to the (π^+ *N*) interaction potential is essentially attributed to the scalar-isoscalar σ and the vector-isovector ρ [5]. According to the interplay between the attractive σ and repulsive ρ mesons, the interaction that is responsible for the cancellation usually happen between these two fields. For pion elastic scattering from a nucleus, the transformed wave function $\psi(r)$ which satisfies a time independent Schrodinger equation was used [8],

$$\{\frac{-\hbar^2}{2\mu}\nabla^2 + U(r)\}\psi(r) = E_{c.m.}\psi(r) , \quad (1)$$

$E_{c.m.}$ is the center-of-mass kinetic energy,

$$E_{c.m.} = \frac{(\hbar k)^2}{2\mu}, \quad (2)$$

k is the wave vector, and a reduced mass μ,

$$\mu = \frac{m_\pi M_t}{m_\pi + M_t} \quad (3)$$

m_π and M_t are the pion and target masses respectively, the effective bombarding pion energy E_L is related to the center-of-mass kinetic energy [9] as follows:

$$E_L = E_{c.m.}\frac{m_\pi + M_t}{M_t} , \quad (4)$$

and the E_L is the energy of the pion related to its relativistic momentum in c.m. system $P_{c.m}$ is as follows [10],

$$P_{c.m} = \frac{M_t P_L}{\sqrt{(m_\pi + M_t)^2 + 2M_t(E_L - m_\pi)}} , \quad (5)$$

P_L is the laboratory momentum of pion.

In equation (1) the total potential $U(r)$ is the sum of two terms: [11],

$$U(r) = V_C(r) + U_F(r), \quad (6)$$

V_C and $U_F(r)$ are the Coulomb potential and pion nucleus potential respectively,

$$V_C(r) = \frac{z_T e^2}{R_C}, \quad (7)$$

Z_T is the target atomic mass number, e^2 = 1.44 MeV fm and R_C is the Coulomb radius [12].

In case of single folded the pion nucleus potential $U_F(r)$ is related to the pion nucleon effective potential $V_{eff(\pi^+ N)}(r)$ as [6].

$$U_F(r) = \int d r_2\, \rho(r_2) V_{eff(\pi^+ N)}(r) , \quad (8)$$

and $\rho(r_2)$ is the distribution of the center of mass of the nucleons in the ground state of the ith nucleus,

$$\rho(r_2) = \rho(r_n) + \rho(r_p), \tag{9}$$

while $\rho(r_n)$ and $\rho(r_p)$ are the neutrons and protons density in the target nucleus, the construction of the densities based on the shell model and in n = p nuclei, proton and neutron densities having the same shape. In Harmonic oscillator (HO) model [13-15],

$$\rho(r_N) = \rho_0\,[1+\alpha(r/a)^2)\exp(-(r/a)^2], \tag{10}$$

where ρ_0 = 0.17± 0.02 fm^{-3} is the nucleon density in the center of the nucleus, and the two parameter α = 2.0 and a =1.746 [13, 16].

Because π^+ has a zero spin and the isospin $\tau_\pi = 1$, then the effective interaction potential may be written as [6],

$$V_{eff(\pi^+ N)}(r) = V_{\pi^+ N}(r) + V_{\pi^+ N}(r)\ \tau_\pi.\tau_N, \tag{11}$$

Where τ_π and τ_N are the isospin for pion and nucleon respectively.

The potential appropriate for the elastic scattering of pion nucleon has been discussed by various researchers [17-22].], so in $\pi^+ N$ interaction,

$$V^{(r)}_{(\pi^+ N)} = V_\sigma(r) + V_\rho(r), \tag{12}$$

After long treatment of this potential as described in a previous study [23], we concluded our realistic optical potential, using the One-Boson-Exchange (OBE) model, as follows:

$$\begin{aligned} V^{(r)}_{(\pi^+ N)} = [\ & V_a + 2C_1 V_d \\ & - C_2\ \frac{d^2}{dr^2} V_a \\ & - C_2\ V_c \frac{d^2}{dr^2} \\ & + 2C_2\, l\,(l+1)\frac{1}{r^2} V_d \\ & - 2C_2\ (S.l\,)\frac{1}{r}\frac{d}{dr} V_b \\ & - 2C_2 \frac{dV_b}{dr}\frac{d}{dr}\ ] \\ & + i[\ 2C_3 (V_d \frac{d}{dr} + \frac{dV_d}{dr})], \end{aligned} \tag{13}$$

Where $V_a(r) = -V_\sigma(r) + V_\rho(r)$,

$V_b(r) = V_\sigma(r) + V_\rho(r)$,

$V_c(r) = V_\sigma(r) + 3V_\rho(r)$,

$V_d(r) = V_\sigma(r) - 2\,V_\rho(r)$,

$C_1 = \frac{E_L^2}{8 m_\pi^2 c^4}$,

$C_2 = \frac{\hbar^2 c^2}{8} \frac{(m_\pi c^2 + M_t c^2)^2}{m_\pi c^2\ M_t c^2}$ and

$C_3 = E_L \frac{\hbar\ c}{8 m_\pi c^2} \frac{(m_\pi + M_t)\ c^2}{m_\pi c^2 M_t c^2}$

The scalar and the vector parts of single meson exchange potential are defined in earlier studies[24-27].

$$V_\sigma(r) = -\frac{g_\sigma^2}{4\pi} J_\sigma(r), V_\rho(r) = \frac{g_\rho^2}{4\pi} J_\rho(r), \tag{14}$$

Where $\frac{g_i^2}{4\pi}$ is the coupling constant and $J_i(r)$ is the generalized Yukawa wave function of the mesons, and i stands for the exchanged mesons σ and ρ.

We use the single particle energy dependant function (SPED) as Yukawa type functions [28], which is given by:

$$J_i(r) = \hbar c \left(\frac{\Lambda_i^2}{\Lambda_i^2 - u_i^2}\right)\left[\frac{\exp(-u_i r)}{r} - \frac{\exp(-\Lambda_i r)}{r}\right] \tag{15}$$

where the parameter $u_i = \frac{m_i c^2}{\hbar c}$ is associated with the masses of the exchanged mesons m_i, $\Lambda_i = \frac{\lambda_i c^2}{\hbar c}$ is associated with the cutoff masses λ_i.

To represent the static meson function, the suggested parameters by the Julich group [5] are presented in Table (1), and the pion, neutron and proton masses are m_π = 139.569 MeV/c^2, m_n = 939.565 MeV/c^2, and m_p = 938.272 MeV/c^2 respectively.

Table (1): The meson masses, coupling constants and the cut-off momentum [5]

Meson	m_i MeV/c^2	$g_i^2/\sqrt{4\pi}$	λ_i GeV/c^2
σ	550	2.385	1.85
ρ	769	0.84	2.4

In case of $\pi^+ N$ system, the wave function of the pion $\varphi_\alpha(\vec{r}_1)$ is normalized because the pion particle has zero spin, but the nucleon wave function $\varphi_\gamma(\vec{r}_2)$ should be converted to the normalized function [19, 27], so the bra wave functions collected by using Clebsch-Gordon and Talmi-Moshinsky-Smirnov (GTMS) coefficients [23, 29-31], then the bra is defined by:

$$\left\langle \varphi_\alpha(\vec{r}_1)\varphi_\gamma(\vec{r}_2)\right| = \sum_{m_{l_\alpha} m_{l_\gamma}, m_{s_\gamma}} (l_\alpha, 0, m_{l_\alpha}, 0 | J_\alpha, m_{J_\alpha})$$
$$(l_\gamma, \tfrac{1}{2}, m_{l_\gamma}, m_{s_\gamma} | J_\gamma, m_{J_\gamma}) \sum_{\lambda\mu} (l_\alpha l_\gamma m_{l_\alpha} m_{l_\gamma} | \lambda\mu)$$
$$\sum_{nl NL} (n_\alpha l_\alpha n_\gamma l_\gamma \lambda) | NLnl\lambda) \sum_{Mm} (Ll Mm|\lambda\mu)$$
$$\sum_{s m_{s_\gamma} T M_T} (0\,\tfrac{1}{2}\,0\,m_{s_\gamma} | s\, m_{s_\gamma})(\tfrac{1}{2}\,\tfrac{1}{2}\,T_\alpha\,T_\gamma\,T\,M_T)$$
$$\left\langle \varphi_{nlm}(\vec{r})\right| \left\langle \varphi_{NLM}(\vec{R})\right| \left\langle \chi^s_{m_{s_\gamma}} \hat{P}_{T_{\alpha\gamma}} \right|, \tag{16}$$

the same treatment is in the Ket term

The differential cross section is given by [32]:

$$\frac{d\sigma}{d\omega}(k,\vartheta) = |f(k,\vartheta)|^2 + |f_c|^2, \tag{17}$$

where $f(k,\vartheta)$ is the scattering amplitude and given by

$$f(k,\vartheta) = \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)[\exp\{i\delta_\ell(k)\}\sin\delta_\ell(k)P_\ell(\cos\vartheta), \tag{18}$$

where $P_\ell(\cos\vartheta)$ is Legendre polynomial, and ϑ is the scattering angle, $|f_c|^2$ is the differential cross section for coulomb scattering potential [32].

$$f_c = \frac{z_1 z_2}{4E_1 Sin^2 \frac{1}{2}\theta} e^2 , \tag{19}$$

In the first Born approximation the phase shift $\delta_\ell(k)$ which is given by,

$$\tan\delta_\ell(k) = -k \left\langle j_\ell(kr)r \middle| U(r) \middle| j_\ell(kr)r \right\rangle \tag{20}$$

where $j_\ell(kr)$ is a spherical Bessel function [32].

Results

The shape of the optical potential

The shape of present optical potential of pion with p, n, ^{16}O and ^{28}Si nuclei are plotted in the flowing figures.

The elastic differential cross section

The elastic differential cross section of $(\pi^+ - {}^{16}O)$ and $(\pi^+ - {}^{28}Si)$ are calculated and compared with the experimental data [33, 34].

Discussion and Conclusion

The realistic optical potential has been previously examined in case of elastic differential cross sections of Kaon meson with nucleon and some nuclei [21], and the theoretical results are fairly good . However, an examination of the elastic differential cross sections of the positive pion on p, n, ^{16}O and ^{28}Si at intermediate energy range of the incident pion by this potential has been conducted.

* In fig. (2), it is clear that the maximum value of the two parts of potential in case of (π^+ n) are higher than that in (π^+ p), this is attributed to the

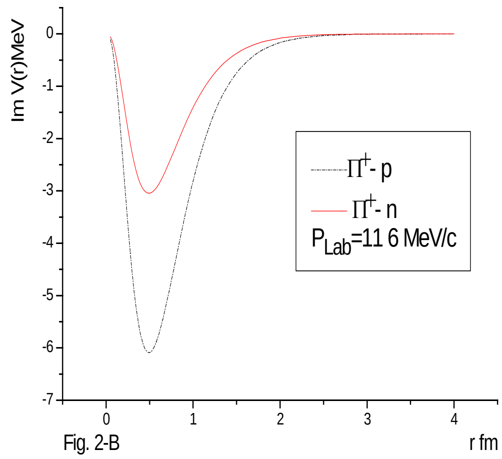
effect of the total isotopic spin for system (π^+ n) and (π^+ p)

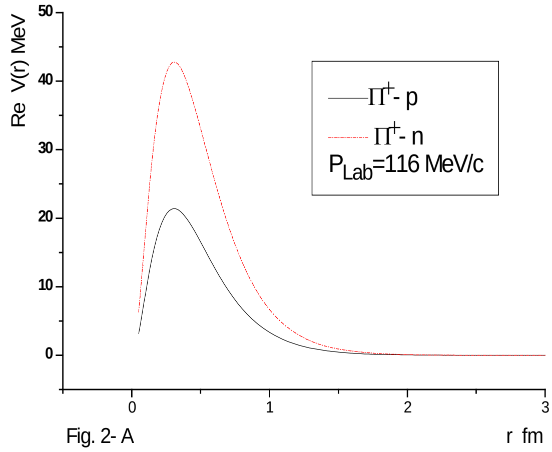
* In figs. (3, 4), in case of ^{16}O or ^{28}Si the absolute value of the real potential is greater than that the imaginary at the same energy, because of the effective field of the repulsive ρ meson mightier than that attractive σ meson. *Also by increasing the mass number of ^{28}Si to be more than that of the ^{16}O, this leads to increasing the values of two parts of the optical potential. This displays the effective of nuclear medium in the scattering process.

* In figs. (5) (π^+ ^{16}O), the theoretical points are in the range of experimental results.

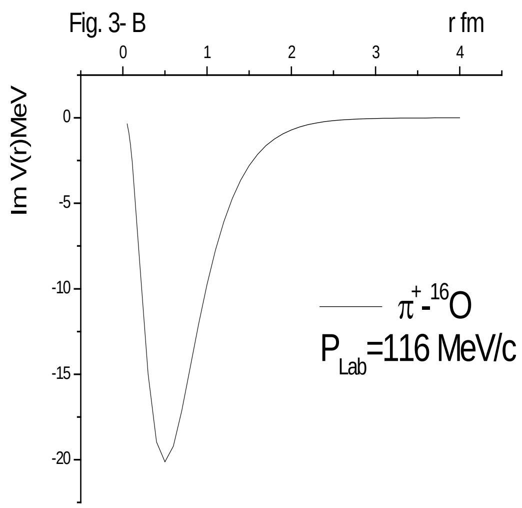
* In figs. (6) (π^+ ^{28}Si), it could be noticed that almost the theoretical results are in the ambit of experimental results with the exception of the points at minimum values.
* In cases of (π^+ ^{16}O) and (π^+ ^{28}Si) figs. (5, 6) the values of the elastic differential cross sections increase by increasing the pion energy at the same angle.

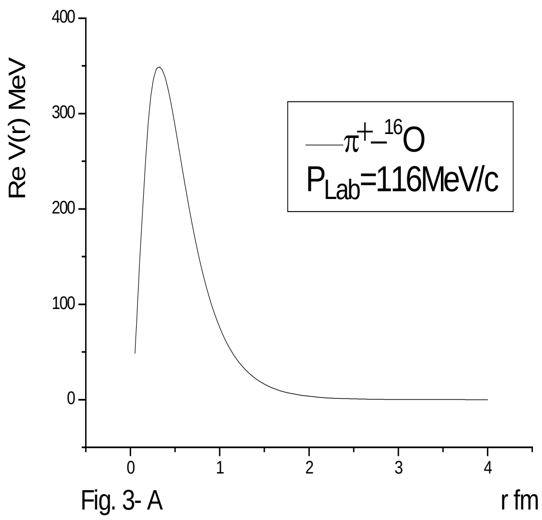
Our theoretical calculations are in agreement, to some extent, with the experimental results, and there are no free parameters in our calculation. This means that the microscopically derived optical potential proved to be a very efficient potentia



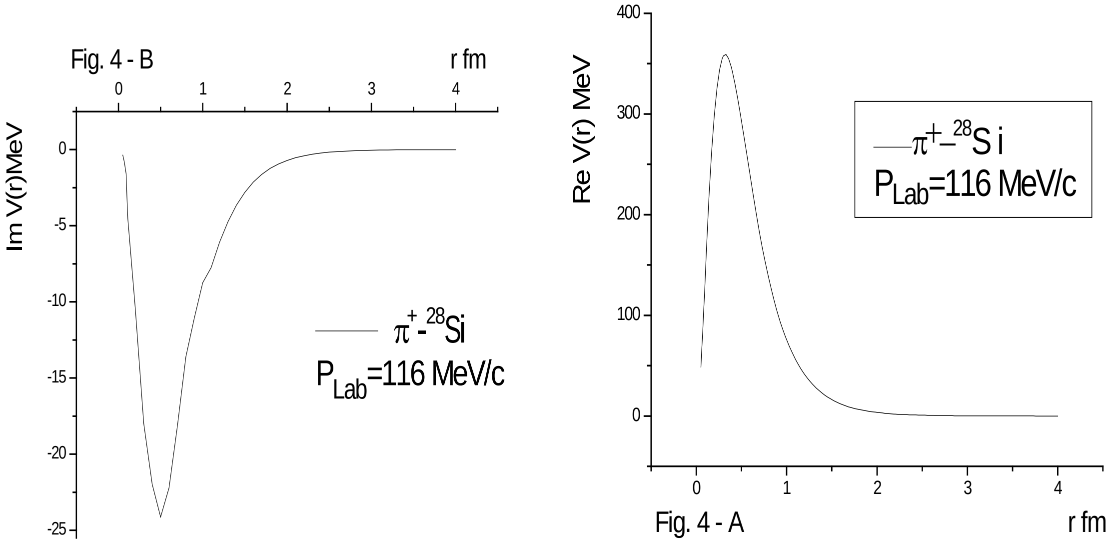


Figs. (2-A, B): The real and imaginary potentials of π^+ n and p are plotted at pion momentum energy 116MeV/c, partial wave L =1 and scattering angle in c.m system is 10.22

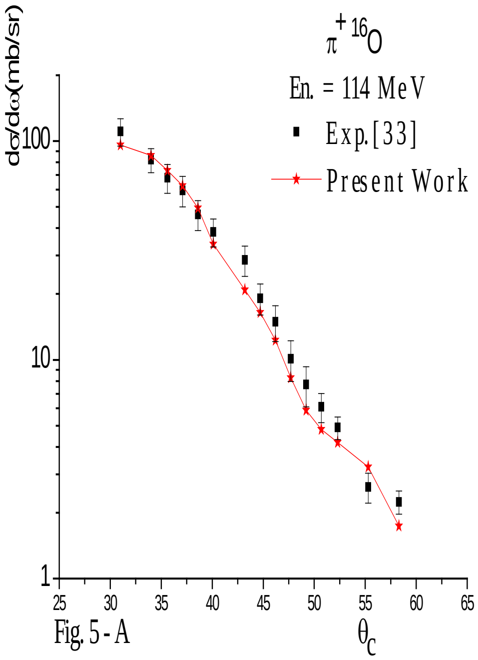


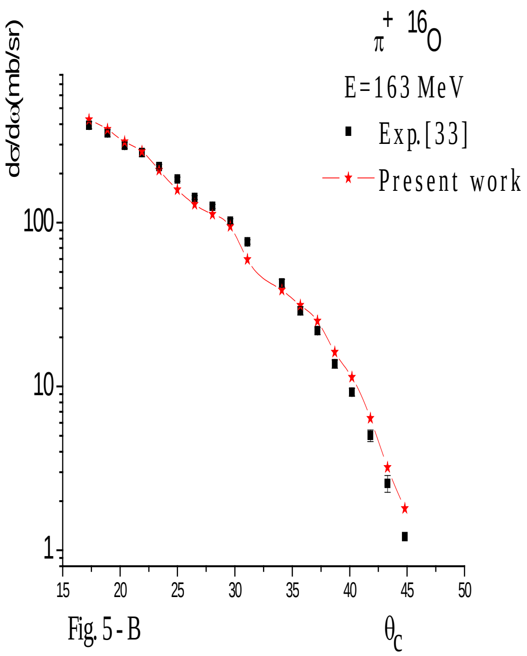


Figs. (3-A, B): The real and imaginary potentials of π^+ ^{16}O plotted at pion momentum energy 116MeV/c, partial wave L =1 and scattering angle in c.m system is 10.22



Figs. (4-A, B): The real and imaginary potentials of π^+ ^{28}Si plotted at pion momentum energy 116MeV/c, partial wave L =1 and scattering angle in c.m system is 10.22





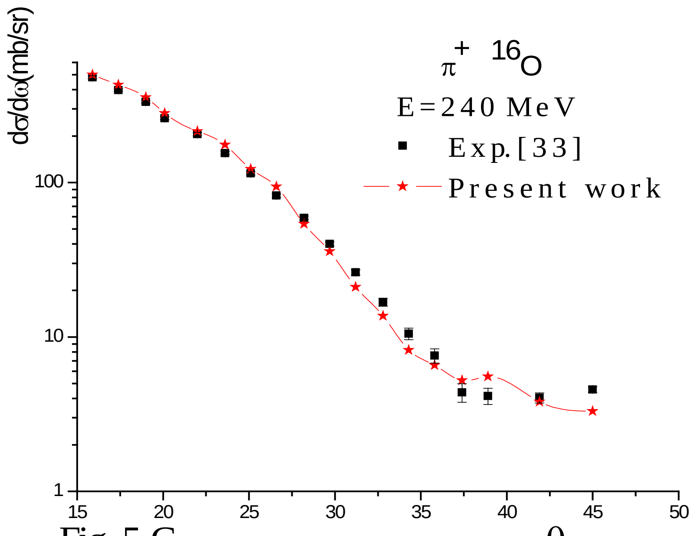


Fig. 5-C

Fig. (5 - A, B, C): The elastic differential cross section for (π^+ ^{16}O) as a function of the scattering angle in the center of mass system

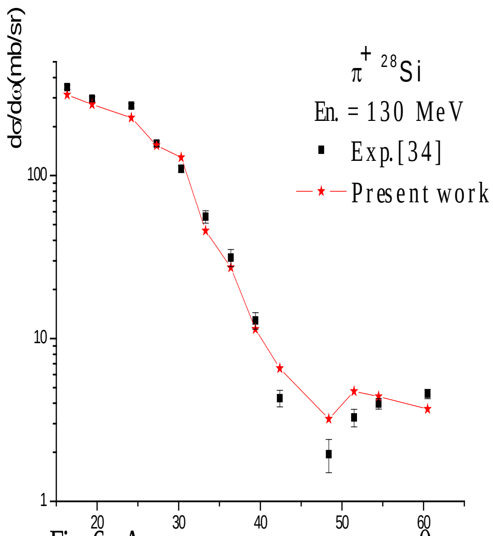


Fig. 6 - A

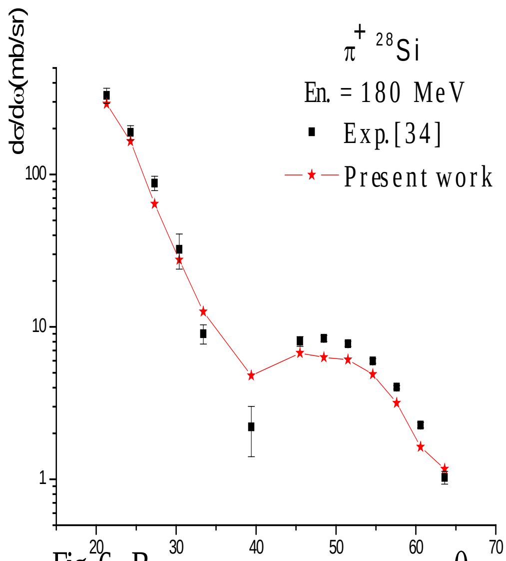


Fig. 6 - B

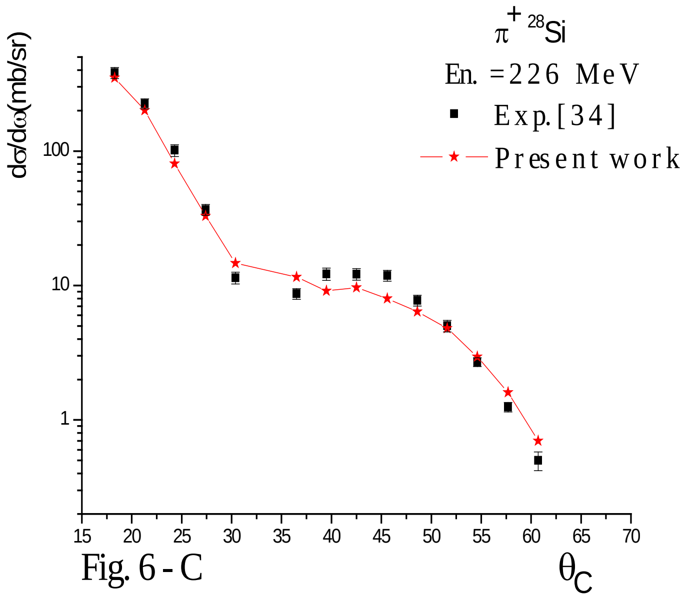


Fig. (6 - A, B, C): The elastic differential cross section for ($\pi^{+}\,{}^{28}Si$) as a function of the scattering angle in the center of mass system.

References

1-Bugg D.V., Gilmore R.S., Knight K.M., Salter D.C., Stafford G.H., Wilson E.J.N., Davies J.D., Dowell J.D., Hattersley P.M., Homer R.J., O'dell A.W., Carter A.A., Tap-per R.J. and Riley K.F., Physical Review 168 . 1466 (1968).

2- Marlow D., Barnes P.D., Colella N.J., Dytman S.A., Eisenstein R.A., Grace R., Takeutchi F., Wharton W.R., Bart S., Hancock D., Hackenberg R., Hungerford E., Mayes W., Pinsky L., Williams T., Chrien R., Palevsky H. and Sutter R., Physical Review. C 25 . 2619 (1982).

3- Chen C.M., Ernst D.J. and Johnson M.B., Physical Review. C 48, 841 (1993).

4- Ebrahim A. A. and Peterson R. J. Physical Review. C 54, No. 5 (1996).

5 -Hoffmann M., Durso J.W., Holinde K., Pearce B.C. and Speth J., Nuclear Physics A 593,341(1995).

6- Sstchler G.R. and Love W.G. Physics. Report 55, No.3 183-254 (1979).

7- Alvarez del Castillo R. and de Takacsy N. B. Physical Review C 43, NO.3 (1991).

8- Khallaf S. A. E. and Ebrahim A. A. Physical Review C 62, 024603 (2000).

9 -Satchler G. R., Nuclear. Physics A 540, 533 (1992).

10- Lukayanov V.K., Zemlyanaya E.V., Lukayanov K.V. and Hana K.M., Physics. of Atomic Nuclei, Vol.73 No. 8 1443(2010).

11- Zuhair F. Shehadeh and Reham M. El-Shawaf , Revista Mexicana de F´ısica 62 475–483 (2016).

12- Reham M. El-Shawaf and Zuhair F. Shehadeh International Journal of Physical 11 85-95 (2016).

13- DE VRIES H., DE JAGER C.W., and DE VRIES C. Atomic and Nuclear data tables 36,795-536 (1987).

14- Augert J.P. et al Nuclear. Physics 12 317 (1986).

15- El-Azab M.F. Physical Review. C 74 064616 (2006).

16- Mikkel B., Johnson and Satchler G.R. Annals of Physics 248, 134 (1996).

17- Holinde K.. Physics. Report. 68, no. 3 (1981).

18- Machleidt R., Holinde K., and Elster Ch.. Physics. Report. 1 149 (1987).

19- Machleidt R., Physical Review. C 63, 024001 (2001).

20- Buttgen R., Holinde K., Müller-Groeling A., Speth J.and Wyborny P., Nuclear Physics A 506. 586 (1990).

21- Green A.E.S. and Sawada T., Review Modern Physics Vol. 39, No. 3 (1967).

22- Keigo Nishimura, Supplement. Progress Theoretical Physics, No. 39, 286 (1967).

23- Hanna K.M., Sewailem Sh.M. and Asmaa G. Shalaby, Arab Journal of Science and Application 49(2), (2016).

24- Sahu B.B., Singh S.k., Bhuyan M., Biswal S.K. and Patra S.K., Physical Review C 89, 034614 (2014).

25 -Miller L.D. and Green A.E., Physical Review C 5, 241(1972).

26- Brockmann R., Physical Review C 18, 1510 (1978).

27- Brockmann R. and Weise W., Physical Review C 16, 1282 (1977).

28- Jaminon M., Mahaux C., and Rochus P., Physical Review C 22. 2027 (1980).

29- R.W.Stagat, F.Riewe, A.E.S.Green, Physical Review C 3. 552 (1971).
30- Brody T. A. and Moshinsky M., Tables of Transformation Brackets, Universidad Nacional Autonoma de Mixico (1960).
31- Smirnov Yu.F., Nuclear Physics.27, 177 (1961). Talmi I. and Helv. Physics Acta 25, 185 (1952).
32- Charles J. Joachain, Quantum Collision Theory, North-Holland ISPN, (1975).
33- Albanes Arieu J.P., Bolger J., Booschitz E. and Ingram C.H.Q. Nuclear Physics. A 350 301 (1980).
34- Preedom B. M., Corfu R., and Egger J. P., et al., Nuclear Physics. A 326, 385-400 (1979).