Microscopic Description of $K^+$ Scattering on H, $^2$H, $^4$He, $^6$Li and $^{12}$C Nuclei Using Meson Exchange Theory

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ABSTRACT

In this research, the total cross sections for the $K^+$ meson interactions with H, $^2$H, $^4$He, $^6$Li, and $^{12}$C nuclei for some incident momenta $P_{lab}$ less than 1 GeV/c was calculated. The $K^+$-nucleon optical potential based on the exchange of three mesons ($\sigma, \rho, \omega$) was derived and also four mesons by adding the ($\sigma_0$) meson between the reactants. It is shown that both the radial behavior of the real and the imaginary parts of the derived potentials in two cases. Comparisons between the available experimental data, other theoretical results and our calculated results showed a reasonable agreement. The extended optical model (by including the $\sigma_0$ meson in the scattering process gave, as expected, results more close to the experimental data. Further, both theoretical and experimental ratios of the total cross sections of the studied nuclei compared with the total cross section with the deuteron nucleus, showed some medium-nuclear effect in such soft nuclear reactions.

Key Words: Optical potential, total cross section, Kaon nucleus scattering.

I - INTRODUCTION

Since the discovery of mesons in cosmic rays (1) and theoretically (2) by proposing the pion meson as the longest component in the nucleon-nucleon force, the theory of meson exchange has extensively developed and, consequently, opened the way to understand better the interaction mechanism between particles and the nuclear forces. The way to that was long and involved many difficulties in putting all the data in a consistent manner. Although, at present, the Quantum Chromo Dynamics (QCD) is believed to be the correct theory for the strong interactions, and there is a lack of a sharp picture for quark-gluon interaction and also the extrapolation of all information coming from different energy domains in one theory (3), we are obliged to use mesons as proper and suitable variables and in the same time as the collective degrees of freedom of QCD theory at low and intermediate energy regions. The meson exchange idea between nucleons was generalized to meson-nucleon and meson-meson interactions supported by the idea that the multi-pion states can be regarded, approximately, as a stable particles with the same quantum numbers (4). One of the distinguished meson exchange models is the Bonn model (5,7).

In the present paper the case study of the scattering of the positive kaon with some light nuclei at intermediate energies is studied. Due to physical nature of the kaon potential (short range and repulsive), we found it is reasonable to drive from the Bonn model the semi-relativistic form of its one-body exchange form. As the kaon is a pseudo-scalar, odd parity, spin zero meson and also transfer to the nucleus an additional quantum number (i.e. the strangeness) and by the weakness nature of the interaction which enable it to penetrate deeper in the nuclear medium, we apply two types of the one-

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body exchange potentials, the first uses 3 exchanged mesons while the second uses 4 exchanged mesons. The outline of the paper is as follows, in Section II a semi-relativistic one body exchange method and the effective interaction was shown. Section III contains results and their discussions. Finally, section IV is devoted to some conclusion.

II - THEORETICAL SCHEME

The most important exchange mesons in the $K^+ N$ interaction are the vector-isovector $\rho (1^-, 1)$, the vector-isoscalar $\omega (1^-, 0)$ and the scalar-isoscalar $\sigma (0^+, 0)$ mesons. According to the interplay between the repulsive $\omega$ and the attractive $\sigma$ mesons, the interaction that responsible for the cancellation usually happened between these two fields.

Then the $K^+ N$ interaction potential according to the One-Boson-Exchange (OBE) model in which $P_{lab} < 1 \, \text{GeV/c}$, $V_{K^+ N}(r)$ can be written as,

$$
V_{K^+ N}(r) = V_\sigma (r) + V_\rho (r) + V_\omega (r)
$$

(1)

Where

$$
V_\sigma (r) = -\gamma^0_K \gamma^0_N J_\sigma (r),
$$
$$
V_\rho (r) = \gamma^0_K \gamma^0_N (\gamma^0_K \gamma^0_N - \vec{\gamma}_K \vec{\gamma}_N) J_\rho (r),
$$
$$
V_\omega (r) = \gamma^0_K \gamma^0_N (\gamma^0_K \gamma^0_N - \vec{\gamma}_K \vec{\gamma}_N) J_\omega (r).
$$

(2)

And

$$
\vec{\sigma} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix}
$$

$\bar{\sigma}$ are the Pauli spin matrices of the nucleon and the $J$'s functions, the positive kaon, $K^+$, is a spinless particle so that we labeled the null matrix ($\vec{\gamma}$) for it, and $J_\sigma (r)$, $J_\rho (r)$ and $J_\omega (r)$ are the Yukawa type exchange - meson functions.

For better description of the theoretical results and towards a reasonable fit with the experimental data, one needs to add an additional repulsive meson in the interaction more than obtained by the $\omega$ meson, a phenomenological repulsive $\sigma_0$-meson (which has shorter range and higher mass than the $\omega$ meson). Then the $K^+ N$ potential will be modified to the form;

$$
V_{K^+ N}(r) = V_\sigma (r) + V_\rho (r) + V_\omega (r) + V_{\sigma_0}(r)
$$

(3)

Where the $\sigma_0$ structure is taken as the structure of the $\sigma$ -meson, but with opposite sign and heavier exchange mass $^{(7)}$ as follows:

$$
V_\sigma (r) = \gamma^0_K \gamma^0_N J_\sigma (r)
$$

(4)
I.1 - The Nucleon and Kaon Wave Functions

The normalization condition for the nucleon wave functions $f_N(r)$ can be written as follows:

$$\langle f_N(\vec{r}) | f_N(\vec{r}') \rangle = \langle \phi_N(\vec{r}) | \phi_N(\vec{r}') \rangle + \langle \chi_N(\vec{r}) | \chi_N(\vec{r}') \rangle$$  \hspace{1cm} (5)

Where, $\phi_2(r)$ and $\chi_2(r)$ are the large and small nucleon wave function components respectively. Consequently, the normalized nucleon wave function can be expressed in the form $^{(13, 14)}$:

$$\left| \phi'_N(\vec{r}) \right| = \sqrt{\frac{2}{m_N c^2}} \left| \phi_N(\vec{r}) \right|$$  \hspace{1cm} (6)

We keep, in this formulation, till the fourth power of relative momentum in the expansion of the square root. Where, $m_N$ is the mass of the nucleon.

One can take approximately only the first term in the expansion of the small wave function component $\chi_N(\vec{r})$ in terms of the large one $\phi_N(\vec{r})$, which is given by the relation $^{(15)}$:

$$\chi_N(\vec{r}) \sim \frac{\vec{\sigma}_N \cdot \vec{P}_N}{2m_N c} \phi_N(\vec{r})$$  \hspace{1cm} (7)

For kaon wave function $\varphi_\alpha(r)$ can be given by;

$$\varphi_\alpha(\vec{r}) = \sum_{m_\ell \alpha} \left( \ell \alpha \ 0 \ m_\ell \alpha \ 0 \right) \varphi_n m_\ell \alpha \ (\vec{r}) \hat{P}_{T\alpha}$$  \hspace{1cm} (8)

Where $\alpha$ represents the quantum number collection in the Clebsch-Gordon coefficient $(n_\alpha, l_\alpha, j_\alpha, m_\alpha)$. The nucleon wave function can be generalized now to be $\varphi_\gamma(\vec{r})$ with subscript $\gamma$ in terms of the spin function $\chi_{m_{\gamma\gamma}}^{1/2}$ and the isotopic spin $\hat{P}_{T\gamma}$ as following,

$$\varphi_\gamma(\vec{r}) = \sum_{m_\ell \gamma, m_{s\gamma}} \left( \ell \gamma \ s \gamma \ m_\ell \gamma \ m_{s\gamma} \right) \varphi_n m_\ell \gamma \ (\vec{r}) \chi_{m_{\gamma\gamma}}^{1/2} \hat{P}_{T\gamma}$$  \hspace{1cm} (9)

Consequently, the $K^+ N$ bra can be rewritten as follows:

$$\left\langle \varphi_\alpha(\vec{r_1}) | \varphi_\gamma(\vec{r_2}) \right\rangle = \sum_{m_\ell \alpha, m_\ell \gamma, m_{s\gamma}} \left( \ell \alpha \ 0 \ m_\ell \alpha \ 0 \right) \left( \ell \gamma \ 0 \ m_\ell \gamma \ 0 \right) \left( \ell \gamma \ 0 \ m_\ell \gamma \ 0 \right) \chi_{m_{\gamma\gamma}}^{1/2} \hat{P}_{T\alpha} \hat{P}_{T\gamma}$$  \hspace{1cm} (10)

The orbital coupling between kaon and nucleon bra can be expressed as,

$$\left\langle \varphi_n \ell \gamma m_\ell \gamma (\vec{r_1}) \varphi_n \ell \gamma m_\ell \gamma (\vec{r_2}) \right\rangle = \sum_{\lambda \mu} \left( \ell \gamma \ 0 \ m_\ell \gamma \ 0 \right) \chi_{m_{\gamma\gamma}}^{1/2} \hat{P}_{T\alpha} \hat{P}_{T\gamma}$$  \hspace{1cm} (11)
The generalized Talmi-Moshinsky-Smirnov (GTMS) brackets for both particles that have different masses in terms of relative and C.M part will be imported (16, 17),

\[
\langle \phi_n^{\alpha_1 \ell_1 \alpha_2 \ell_2} | N L n \ell \lambda_1 \lambda_2 | \phi_{N L n}^{\alpha \lambda \mu} (\vec{r}_1, \vec{r}_2) \rangle = \sum_{n(NL)} (n \alpha_1 \ell_1 \alpha_2 \ell_2 \lambda_1 \lambda_2) | N L n \ell \lambda | \langle \phi_{N L n}^{\alpha \lambda \mu} (\vec{r}_1, \vec{r}_2) \rangle \tag{12}
\]

Consequently, the wave function of \( K^+ N \) system can be separated into two components, the wave function of the relative motion \( \vec{r} \) and the motion represents the C.M. \( \vec{R} \) as follows:

\[
\langle \phi_{N L n}^{\alpha \lambda \mu} (\vec{R}, \vec{r}) | \phi_{N L n}^{\alpha \lambda \mu} (\vec{R}, \vec{r}) \rangle = \sum_{M m} (L \ell Mm) | \phi_{N L M}^{\alpha \lambda \mu} (\vec{R}) \rangle | \phi_{N L m}^{\alpha \lambda \mu} (\vec{r}) \rangle \tag{13}
\]

Moreover, the spin and isotopic spin nucleon wave functions can be expanded as follows:

\[
\langle \hat{T}_T^{1/2} m_{s_{\gamma} \gamma}^{1/2} \hat{T}_T^{1/2} | \phi_{N L n}^{\alpha \lambda \mu} (\vec{R}, \vec{r}) \rangle = \sum_{m_{s_{\gamma} \gamma}^{1/2} T M T} \left( 0 \frac{1}{2} m_{s_{\gamma} \gamma}^{1/2} | s m_{s_{\gamma} \gamma}^{1/2} \right) \left( \frac{1}{2} T_{\alpha T_{\gamma}} T_{M T} \right) \langle \chi_{m_{s_{\gamma} \gamma}^{1/2}}^{s} \hat{P}_{T_{\alpha \gamma}} \rangle \tag{14}
\]

**II.2- Kaon-Nucleon and Nucleus Potentials**

After some arrangements for the derived optical potential (real and imaginary) parts. We obtained the potential in the form:

\[
V_{R(k^+ N)}(r) = V_A + 4C_1 V_D + C_2 \left[ 4\ell(\ell + 1) \frac{1}{r^2} V_D + 4\ell S \frac{dV_B}{dr} + 2 \frac{dV_B}{dr} \frac{d}{dr} + \frac{d^2V_A}{dr^2} - V_C \frac{d^2}{dr^2} \right]
\]

\[
V_{Im(k^+ N)}(r) = i4C_3 \left( \frac{dV_D}{dr} + V_D \frac{d}{dr} - \frac{2}{\hbar} \ell S \frac{1}{r} V_B \right) \tag{15}
\]

where, \( C_1 = E_1^2 \frac{1}{m_1^2} \frac{1}{c^4} \), \( C_2 = \frac{\hbar^2 c^2}{8} \frac{(m_1 c^2 + m_2 c^2)^2}{m_1 c^2 m_2 c^2} \) and \( C_3 = \frac{\hbar c E_1}{8} \frac{(m_1 + m_2)^2 c^2}{m_2 c^2 m_1^2 c^4} \)

\( m_1 \) and \( m_2 \) are the kaon and nucleon masses respectively, and \( E_1 \) is the total energy of the kaon. the relativistic momentum of a kaon in c.m. system \( P_{c.m} \) is as follows[18]

\[
P_{c.m} = \frac{m_2 P_{Lab}}{\sqrt{(m_1 + m_2)^2 + 2m_2(E_1 - m_1)}}
\]

\[
V_A(r) = -V_\sigma(r) + V_\rho(r) + V_\omega(r) + V_{\sigma_0}(r),
\]

\[
V_B(r) = V_\sigma(r) + V_\rho(r) + V_\omega(r) - V_{\sigma_0}(r),
\]

\[
V_C(r) = 3V_\rho(r) + 3V_\omega(r) - V_{\sigma_0}(r),
\]

\[
V_D(r) = V_\rho(r) + V_\omega(r) \tag{16}
\]
For simplicity, to calculate the many body interacting potential $V_{K^+A}(r)$, we use the approximated form as the sum of the $K^+N$ potentials, $V_{K^+A}(r)$ as follows:

$$V_{K^+ A}(r) \approx \sum_{N_i=1}^{A} V_{K^+ N_i}(r)$$

(17)

The reduced potential can be calculated as [26],

$$U_{K^+ A}(r) = \frac{2\mu}{\hbar^2} V_{K^+ A}(r)$$

(18)

Therefore phase shift can be calculated, then one can calculate the total cross section, see details in (19).

### III. RESULTS AND DISCUSSIONS

In the present work, we have adopted the associated generalized Yukawa (GY) meson function (20) which is given by,

$$J_i(r) = g_i^2 \hbar c \left[ \frac{\exp(-u_i r)}{r} - \frac{\exp(-\Lambda_i r)}{r} \left(1 + \frac{\Lambda_i^2 - u_i^2}{2\Lambda_i} \right) \right]$$

(19)

Where the parameter $u_i = \frac{m_i c^2}{\hbar c}$ is associated with the masses of the exchanged mesons $m_i$, $g_i$ its coupling constants, $\Lambda_i = \frac{\lambda_i c^2}{\hbar c}$ the cutoff parameter associated with the cutoff masses $\lambda_i$, and $i$ stands for the exchanged particles.

In our study for the interaction of the kaon with nucleons and nuclei, we have taken the parameters suggested by the Julich group (7). Table (1) shows the different kinds of mesons that exchange during the interaction, $\sigma, \rho, \omega$ and the additional supposed meson $\sigma_0$. In the present work we have studied the optical potential for 3 and 4-meson exchange but the total cross section was calculated at the 4-meson exchange. The kaon and nucleon masses are taken, as $m_1 = 495.82$ MeV/c$^2$, and $m_2 = 938.926$ MeV/c$^2$ respectively.

| Table (1) | The exchanged meson masses, coupling constants and the cut-off parameters. |
|-----------|-----------------|-----------------|-----------------|
| meson    | $m_i$ MeV/c$^2$ | $g_i \sqrt{4\pi}$ | $\lambda_i$ GeV/c$^2$ |
| $\sigma$ | 600             | 1.300            | 1.7             |
| $\rho$   | 769             | 0.773            | 1.4             |
| $\omega$ | 782.6           | 2.318            | 1.5             |
| $\sigma_0$ | 1200          | -40              | 1.5             |
In Fig.(1-a, b) calculations for the $K^+$ proton real and imaginary parts of our two constructed optical potentials based on the exchange of 3- and 4-mesons between the kaon and the nucleon are shown. It is noticed that the real parts (3M, 4M) of the potentials give maximum value at a distance of 0.5 Fermi, 8.5 MeV in case of the 4M exchange model, and for $K^+$-proton about 5.5 MeV in 3M exchange model. In fig. 1(b) with respect to the imaginary part, the absorption reaches, to about (-2.5) MeV and (-1.75 MeV) respectively, at the same distance.

Fig.(2) shows the calculated total cross section of the $K^+\text{p}$ compared with the experimental data. We noticed that our theoretical results based on the both three and four meson exchange (3M-4M) models, the latter gives close results to the experimental data.

Figs.(3- a, b) shows the real and imaginary parts of the optical potential used for both 3M and 4M-models in the interaction $K^+\text{2H}$. We noticed that the maximum or the minimum values for the real and imaginary parts are about 15, 18 MeV and (-7), (-9) MeV for the 3M- and 4M- models respectively.

Fig.(4) illustrates the theoretical results for the total cross section for the $K^+\text{2H}$ interaction as compared with the experimental data. The figure shows that the theoretical 4M-model fits better the experimental results.

Fig.(5- a, b) interprets the real and imaginary parts of the optical potential in $K^+\text{4He}$ interaction. We found that the maximum (minimum) values are 30, 35 MeV and (-14, -18) for the 3M- and 4M- models respectively.

Fig. (6) shows the calculated total cross section in comparison with the theoretical results of $(22)$. It is noticed that the present theoretical 4M- model matches very well the previously reported data.

Fig. (7) reveals the ratio of the total cross section for the $\text{4He}$ nucleus to its corresponding value of $\text{3H}$ nucleus for the 4M- theoretical model in comparison with the theoretical result of Jaing $(24)$. Using our 4 M-theoretical model the ratios deviate from unity with values between 0.93 to 0.99, while in ref. $(24)$ the ratios are ranges from 0.94 to 0.89 in the momentum range 400-700 MeV. These theoretical ratio results prove that there exists an EMC-phenomenon (European Muon Collaboration) at this momentum range.

Fig.(8- a, b) shows the real and imaginary parts of the optical potential of $K^+\text{6Li}$ scattering. The maxima of the real part are at about 0.5 fm having the values of 35 and 45 MeV for the 3M- and 4M- models respectively. The minima for the imaginary parts have the values of (-20) MeV for the 3M- and 4M- models respectively.

Fig. (9) demonstrates the total cross section calculated theoretically using our model and comparison with different experimental data were given. The 4M- theoretical model has clear close predictions to the experimental total cross sections than the 3M-model.

In Fig. (10) the experimental ratio wave around unity which prove that there exist some in-medium effect, while in the theoretical results show also some values due to the in-medium effect.

In the case of $K^+\text{12C}$ scattering, we gave in Fig. (11-a, b) the real and imaginary parts of the potential where it was found there maximum (minimum) values 45,55 MeV and (-20, -27) MeV for 3M- and 4M-models respectively.

In Fig. (12) the theoretical and different experimental total cross sections were plotted and we notice a good description of the 4M-model with respect to the experimental data.

In Fig. (13) the experimental and the theoretical ratios were plotted and it is evident that there is in-medium effect and, as expected, the advantage of the 4M model.
Fig. 1 (a, b). The real and imaginary parts of the optical potential for 3-and 4–exchanged mesons in case of $K^+ p$ interaction at $P_{lab}=450$ MeV.

Fig. (2): The total cross section versus the momentum in $K^+ p$ reaction.

Fig. 3(a, b): The real and imaginary parts of the optical potential for 3 and 4–exchanged mesons in case of $K^+ ^2H$ interaction at $P_{lab}=450$ MeV.
Fig. (4): The total cross section versus the momentum in $K^+^4He$ reaction.

Fig. 5(a, b): The real and imaginary parts of the optical potential for $3$ and $4$ exchanged mesons in case of $K^+^4He$ interaction at $P_{lab}=450$ MeV.

Fig. (6): The total cross section versus the momentum in $K^+^4He$ reaction.
Fig. (7): The Ratio of Helium to Deuterium cross sections per nucleon. The solid square symbols present theoretical calculations and the dashed line indicate Jaing results (24).

Fig.8 (a, b): The real and imaginary parts of the optical potential for 3 and 4 – exchanged mesons in case of $^{6}\text{Li}$ interaction at $P_{lab}=456$ MeV.

Fig. (9): The total cross section versus the momentum in $^{6}\text{Li}$ reaction.
Fig. (10): The Ratio of Lithium to Deuterium theoretical cross sections per nucleon. The solid square symbols indicate the theoretical and the experimental ratio represented by Spherical, Weiss et al. (25).

Fig. (11): (a, b). The real and imaginary parts of the optical potential for 3 and 4 – exchanged mesons in case of $K^+ {^{12}C}$ interaction at $P_{lab}=456$ MeV.

Fig. (12): The total cross section versus the momentum in $K^+ {^{12}C}$ reaction.
V. CONCLUSION

In the present work the total scattering of the positive Kaon on H, 2H, 4He, 6Li and 12C have been studied at intermediate energy range of the incident Kaon. Important conclusion can be derived from this study as follows:

1. The microscopically derived optical potential used in our calculations, which based on the one-boson-exchange method to describe the reaction between the Kaon-reactant, proved itself a very efficient potential.

2. The potential we derived and applied have two essential features, the repulsive and the short range characters. These features are required and consistent with the experimental data.

3. The above two features of the potential encourage us to suffice in our calculations with the impulse approximation, which showed reasonable agreement with the data at the appointed energies.

4. The nuclear medium effects and the charge dependence were reported in our calculations via the wave vector and the Clebsch-Gordon coefficients. The charge dependence has been accounted where in our calculations the interaction of the Kaon with the two different isotopic spin forms of the nucleon give different values for the Clebsch-Gordon coefficients.

5. The spin-orbit force was included in the calculations without using any further adjustable parameters.

6. Due to that the data needs more repulsion and the \( \omega \) meson can not bear alone the burden of that, we suggest the use of the \( \sigma_0 \) meson with heavier mass than the sigma meson and opposite sign to account for the additional repulsion required by the data. In fact, as expected, the calculated results of the total cross section gave more consistency with the measured data.

7. In the present work, and by comparing the theoretical results with the experimental data, we notice the existence of medium effect phenomena in these soft nuclear reactions.

8. Here we have used the more accurate total values of Clebsch-Gordon coefficients for each shell, instead of nuclear density which used in the folding method.

9. The theoretical results confirm the suggest adding the \( (\sigma_0) \) meson.

This work urge us to continue this study to get more accurate theory approach. And to investigate the \( K^+ \) (the most weak probe in the strong interaction theory)- nucleus interaction in the near future.
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